TRANSIENT ANALYSIS IN PIPE NETWORKS

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Abstract

Power failure of pumps, sudden valve actions, and the operation of automatic control systems are all capable of generating high pressure waves in domestic water supply systems. These transient conditions resulting in high pressures can cause pipe failures by damaging valves and fittings. In this study, basic equations for solving transient analysis problems are derived using method of characteristics. Two example problems are presented. One, a single pipe system which is solved by developing an excel spreadsheet. Second, a pipe network problem is solved using transient analysis program called TRANSNET.

A transient analysis program is developed in Java. This program can handle suddenly-closing valves, gradually-closing valves, pump power failures and sudden demand changes at junctions. A maximum of four pipes can be present at a junction. A pipe network problem is solved using this java program and the results were found to be similar to that obtained from TRANSNET program. The code can be further extended, for example by developing java applets and graphical user interphase to make it more user friendly.

A two dimensional (2D) numerical model is developed using MATLAB to analyze gaseous cavitation in a single pipe system. The model is based on mathematical formulations proposed by Cannizzaro and Pezzinga (2005) and Pezzinga (2003). The model considers gaseous cavitation due to both thermic exhange between gas bubbles and surrounding liquid and during the process of gas release. The results from the model show that during transients, there is significant increase in fluid temperature along with high pressures. In literature pipe failures and noise problems in premise plumbing are attributed to gaseous cavitation.

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Notation

The following symbols are used in this report:

A Cross sectional area of the pipe

C Courant number

C_a Concentration in gas phase

C_P Specific heat of air at constant pressure

C_v Specific heat of air at constant volume

C_w Molar concentration (mol / L) of the dissolved gas or aqueous phase molar concentration

c Wave speed of pure liquid in an elastic pipe

E Bulk modulus of the liquid

H Piezometric head

h Enthalpy per unit mass

K_H Henry's constant $(atm.L / mol)$

K_N Experimental positive constant $(1/\sec)$

k Ratio between specific heat at constant pressure and constant volume

M Molecular mass of the gas

m Mass of free gas per unit volume

n Number of moles

p Absolute pressure

p_g Gas pressure above liquid at the beginning of evolution or solution process

p_l Partial pressure exerted by the liquid

p_s Equilibrium or saturation pressure

p_v Vapor pressure

Q Discharge in the pipe

q Heat transfer per unit mass and time

R Universal gas constant

R_0 Pipeline radius

$\Re$ Initial Reynolds number

r Distance from the axis

T_0 Absolute liquid temperature assumed to be constant

S Gas entropy

U Internal energy

u Velocity component in longitudinal direction

u_* Friction velocity

V Volume of gas

V_l Volume of liquid

V_V Volume of gas

y Distance from wall

z Elevation head

α Void fraction

ρ_g Density of gas

ρ_l Density of liquid

σ_x Normal stress in longitudinal direction

σ_r Normal stress in radial direction

σ_θ Normal stress in angular direction

τ Wall shear stress

τ_w Wall shear stress

ν Kinematic viscosity

Chapter 1: Introduction

Devices such as valves, pumps and surge protection equipment exist in a pipe network. Power failure of pumps, sudden valve actions, and the operation of automatic control systems are all capable of generating high pressure waves in domestic water supply systems. These high pressures can cause pipe failures by damaging valves and fittings. Study of pressure and velocity variations under such circumstances is significant for placement of valves and other protection devices. In this study, the role of each of these devices in triggering transient conditions is studied. Analysis is performed on single and multiple pipe systems.

Transient analysis is also important to draw guidelines for future pipeline design standards. These will use true maximum loads (pressure and velocity) to select the appropriate components, rather than a notional factor of the mean operating pressure. This will lead to safer designs with less over-design, guaranteeing better system control and allowing unconventional solutions such as the omission of expensive protection devices. It will also reveal potential problems in the operation of the system at the design stage, at a much lower cost than during commissioning.

1.1 Literature Review

Most of the problems considering unsteady pure liquid flow in pipes are solved using a set of partial differential equations (Wylie and Streeter, 1993) which are discussed in detail in Chapter 2. These equations are valid only when the pressure is greater than the vapor pressure of liquid, and are solved numerically using the method of characteristics which was introduced by Streeter and Wylie (1967). But in many flow regimes, small amount of free gas is present in a liquid. When local pressure during transient drops below saturation pressure, the liquid releases free gas. If the pressure drops to vapor pressure, cavities are formed (Tullis et al., 1976). The former occurrence is called gaseous cavitation where as the latter occurrence is called vaporous cavitation.

Martin et al., (1976) developed a one-dimensional homogeneous bubbly model using a two step Lax-Wendroff scheme. Pressure wave propagation and interactions are handled well by Lax-Wendroff scheme by introducing a pseudo-viscosity term. The results produced by this model compare favorably with experiment than by using fixed grid method of characteristics. An analytical model was developed by Wiggert and Sundquist (1979) to investigate gaseous cavitation using the method of characteristics. Gas release is assumed mainly due to difference in local unsteady pressure and saturation pressure. Increase in void fraction due to latent heat flow is not considered. Wylie (1984) investigated both gaseous and vapor cavitation using a discrete free gas model. Free gas is lumped at discrete computing locations and pure liquid is assumed in between these locations. Small void fraction and isothermal behavior of fluid are some of the assumptions made. Gaseous cavitation is simulated and it gave close results when compared with other methods. Pezzinga (1999) developed a 2D model, which computes frictional losses in pipes and pipe networks using instantaneous velocity profiles. The extreme values for pressure heads and pressure wave oscillations were well reproduced by this model.

Pezzinga (2004) adopted "second viscosity" to better explain energy dissipation during transient gaseous cavitation. Constant mass of free gas is assumed at constant temperature. Second viscosity or bulk viscosity coefficient accounts for other forms (other than frictional losses) of energy dissipation such as gas release and heat exchange between gas bubbles and surrounding liquid.

Cannizzaro and Pezzinga (2005) considered the effects of thermic exchange between gas bubbles and surrounding liquid and gas release and solution process separately to study energy dissipation during gaseous cavitation. Separate 2D models were considered. The results of numerical runs shows that 2d model with gas release allows for a good simulation of the experimental data.

1.2 Objective

The objectives of this research are to:

1) Study unsteady flow in pipes and pipe networks carrying pure liquid, including evaluation of pressure and velocity heads at nodes and junctions at different time intervals,
2) Develop a program using object oriented technology to analyze transients in pipes considering single phase flow, and
3) Study the effects of gaseous cavitation on fluid transients using equations developed by Cannizzaro and Pezzinga (2005).

1.3 Organization

This thesis is divided into five chapters. Chapter 1 includes a brief introduction to transients, review of literature, and objectives of the study. Basic equations of transient flow analysis in pipe networks are discussed in Chapter 2. Two example problems are solved using excel spreadsheet to demonstrate the method of characteristics. Chapter 3 is devoted to use of object oriented technology for analyzing transient problems in a pipe network. Comparison is drawn between procedural language and object oriented approach of analyzing transients in a pipe network. Chapter 4 is about gaseous cavitation in pipes where energy dissipation due to gas release and solution process is studied. Here, thermal exchange between gas bubbles and surrounding liquid is also considered. A comprehensive model to obtain the amount of gas release is developed. Chapter 5 presents the summary of work presented in this thesis, and also discusses its potential application.

Chapter 2: Basic Equations of Transient Flow Analysis in Closed Conduits

2.1 Introduction

Initial studies on water hammer are done assuming single phase flow of fluid (Wylie et al., 1993). The method of characteristics is most widely used for modeling water hammer. First, the fundamental equations involved in water hammer analysis are discussed, following which two example problems are solved to highlight the analytical technique.

2.2 Unsteady Flow Equations

Study of transient flow includes fluid inertia and also elasticity or compressibility of the fluid and the conduit. The analysis of transient flow in either of these cases requires the application of Newton's second law which leads to the Euler equation as discussed below.

2.2.1 The Euler Equation

Consider a small cylindrical control volume of fluid at the pipe centerline as shown in figure 1.

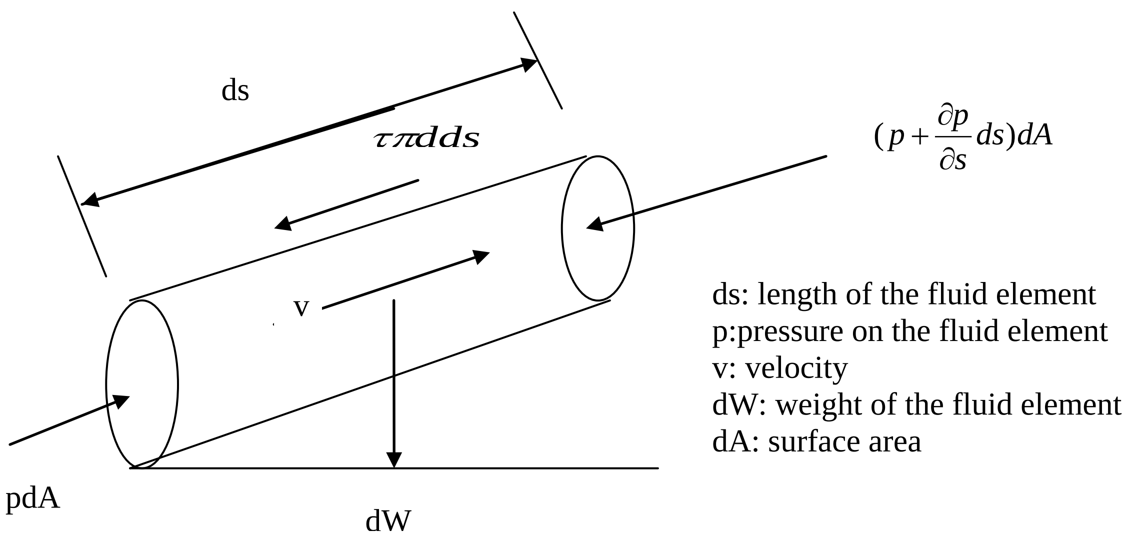


Figure 1. Cylindrical fluid element with all forces shown.

The Euler equation (Wylie, et al.,1993) is derived by applying Newton's second law to this control volume.

$$\sum F_s = ma_s = m\frac{dv}{dt} \tag{2.1}$$

Where m is the fluid mass and $\frac{dv}{dt}$ is the total derivative of the fluid velocity.

Substituting the applied forces into Eq. 2.1., and writing mass in terms of density and volume results in

$$pdA - \left(p + \frac{\partial p}{\partial s} ds \right) dA - dW \sin\theta - \tau\pi d(ds) = \frac{dW}{g}\frac{dv}{dt} \tag{2.2}$$

If we divide by dW and on simplification yields

$$-\frac{\partial p}{\partial s} dsdA \frac{1}{dW} - \sin\theta - \tau\pi d(ds)\frac{1}{dW} = \frac{1}{g}\frac{dv}{dt} \tag{2.3}$$

We can further substitute $\sin\theta = \frac{\partial z}{\partial s}$ and $dW = \gamma ds(dA)$ which yields:

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \tau\pi d \frac{1}{\gamma(dA)} = \frac{1}{g}\frac{dv}{dt} \tag{2.4}$$

on further substituting of $dA = \frac{\pi d^2}{4}$ we get:

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau}{\gamma d} = \frac{1}{g}\frac{dv}{dt} \tag{2.5}$$

If we expand the cross section of the element to fill the pipe cross section and introduce average velocity V, we obtain:

$$-\frac{1}{\gamma}\frac{\partial p}{\partial s} - \frac{\partial z}{\partial s} - \frac{4\tau_0}{\gamma D} = \frac{1}{g}\frac{dV}{dt} \tag{2.6}$$

Where: τ_0 is the shear stress at the wall. Larock et al., (2000) expressed τ_0 as:

$$\tau_0 = \frac{1}{8} f\rho V|V| \tag{2.7}$$

Where f is the Darcy-Weisbach friction factor, ρ is the density of the fluid and V is its velocity.

After substituting Eq. 2.7 in Eq. 2.6 and under the assumption that the local elevation of the pipe can be described solely as a function of location s, we obtain the Euler equation of motion:

$$\frac{dV}{dt} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\frac{dz}{ds} + \frac{f}{2D}V|V| = 0$$

Or,

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s} + \frac{1}{\rho}\frac{\partial p}{\partial s} + g\sin\alpha + \frac{f}{2D}V|V| = 0 \qquad (2.8)$$

2.2.2 Conservation of Mass

Applying conservation of mass to a control volume that coincides with the interior of the pipe and is of length ds: See figure 2.

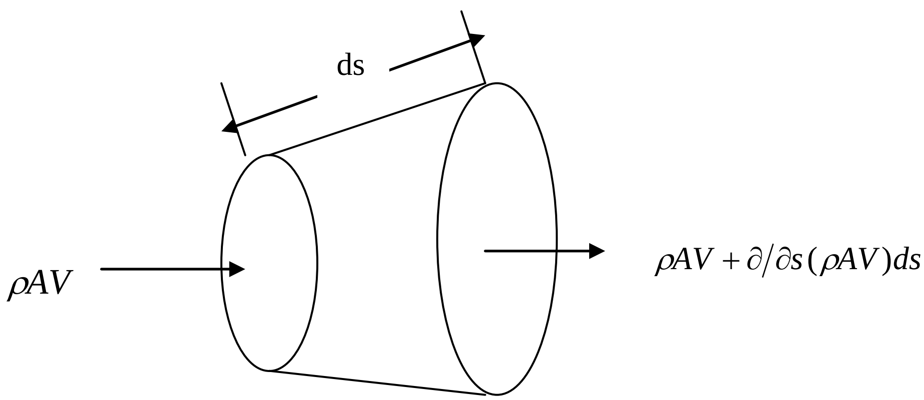


Figure 2. Control volume coinciding with the interior surface of the pipe

In above figure, ρ is the density of the fluid, A is the cross section area of control volume and V is the velocity of the fluid.

From Wylie, et al., (1993):

$$\rho AV - \left[\rho AV + \frac{\partial}{\partial s}(\rho AV)ds\right] = \frac{\partial}{\partial t}(\rho Ads) \qquad (2.9)$$

Or $$-\frac{\partial}{\partial s}(\rho AV)ds = \frac{\partial}{\partial t}(\rho Ads) \qquad (2.10)$$

Expanding the parenthesis of Eq. 2.10 yield:

$$-\left(\rho A\frac{\partial V}{\partial s}ds + \rho V\frac{\partial A}{\partial s}ds + AV\frac{\partial \rho}{\partial s}ds\right) = \rho A\frac{\partial}{\partial t}(ds) + \rho ds\frac{\partial A}{\partial t} + Ads\frac{\partial \rho}{\partial t} \qquad (2.11)$$

Dividing both sides by the control volume mass ρAds,

$$-\left(\frac{\partial V}{\partial s}+\frac{1}{A}V\frac{\partial A}{\partial s}+\frac{1}{\rho}V\frac{\partial \rho}{\partial s}\right)=\frac{1}{ds}\frac{\partial}{\partial t}(ds)+\frac{1}{A}\frac{\partial A}{\partial t}+\frac{1}{\rho}\frac{\partial \rho}{\partial t} \tag{2.12}$$

Regrouping above Eq we get:

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial t}+V\frac{\partial \rho}{\partial s}\right)+\frac{1}{A}\left(\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial s}\right)+\frac{1}{ds}\frac{\partial}{\partial t}(ds)+\frac{\partial V}{\partial s}=0 \tag{2.13}$$

Recognizing that $\frac{\partial \rho}{\partial t}+V\frac{\partial \rho}{\partial s}=\frac{d\rho}{dt}$ and $\frac{\partial A}{\partial t}+V\frac{\partial A}{\partial s}=\frac{dA}{dt}$, Eq. 2.13 becomes

$$\frac{1}{\rho}\frac{d\rho}{dt}+\frac{1}{A}\frac{dA}{dt}+\frac{1}{ds}\frac{d}{dt}(ds)+\frac{\partial V}{\partial s}=0 \tag{2.14}$$

^ ^ ^ ^

Terms: **T(1)** **T(2)** **T(3)** **T(4)**

Let's split this Eq. 2.14 in various terms as below, we get:

T(1): $\frac{1}{\rho}\frac{d\rho}{dt}$

Bulk modulus of elasticity for a liquid (K) is expressed as (Larock et al., 2000):

$K=-\frac{dp}{d\nu/\nu}=\frac{dp}{d\rho/\rho}$, Now **T(1)** becomes:

$$\frac{1}{\rho}\frac{d\rho}{dt}=\frac{1}{K}\frac{dp}{dt} \tag{2.15}$$

T(2): $\frac{1}{A}\frac{dA}{dt}$

The above term can be expressed as (Larock et al., 2000):

$$\frac{1}{A}\frac{dA}{dt}=(1-\mu^2)\frac{D}{eE}\frac{dp}{dt} \tag{2.16}$$

Where,

μ = Poisson's ratio (of the pipe)

e = Pipe wall thickness

E = Young's Modulus of the pipe

T(3): $\frac{1}{ds}\frac{d}{dt}(ds)$

Considering longitudinal expansion of the pipe (Larock et al., 2000):

$$d(ds) = d\varepsilon_1 ds \tag{2.17}$$

Where ε_1 is the strain along the pipe axis.

For all the buried pipes, axial movement is restrained. So differential change in strain along pipe axis ($d\varepsilon_1$) = 0.

Therefore, $\frac{1}{ds}\frac{d}{dt}(ds) = 0$

Making all these substitutions in Eq. 2.14 yields,

$$\frac{1}{K}\frac{dp}{dt} + (1-\mu^2)\frac{D}{eE}\frac{dp}{dt} + \frac{\partial V}{\partial s} = 0 \tag{2.18}$$

$$\frac{dp}{dt}\left(\frac{1}{K} + (1-\mu^2)\frac{D}{eE}\right) + \frac{\partial V}{\partial s} = 0 \tag{2.19}$$

Wave speed (Larock et al., 2000) can be defined as the time taken by the pressure wave generated by instantaneous change in velocity to propogate from one point to another in a closed conduit. Wave speed(c) can be expressed as:

$$c^2\rho\left[\frac{1}{K} + \frac{D}{e}\left(\frac{1-\mu^2}{E}\right)\right] = 1 \tag{2.20}$$

Hence by using Eq. 2.20 we can write Eq. 2.19 as,

$$\frac{dp}{dt}\left(\frac{1}{c^2\rho}\right) + \frac{\partial V}{\partial s} = 0 \tag{2.21}$$

$$\frac{dp}{dt} + c^2\rho\frac{\partial V}{\partial s} = 0 \tag{2.22}$$

Or,

$$\frac{\partial p}{\partial t} + V\frac{\partial p}{\partial s} + c^2\rho\frac{\partial V}{\partial s} = 0 \tag{2.23}$$

2.3 Method of characteristics

The significance of method of characteristics is the successful replacement of a pair of *partial* differential equations by an equivalent set of *ordinary* differential equations. The method of characteristics is developed from assuming that the Eq. 2.9 and Eq. 2.23 can be replaced by a linear combination of themselves (Wylie, et al.,1993)

$$\lambda\left[\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial s}+\frac{1}{\rho}\frac{\partial p}{\partial s}+g\sin\alpha+\frac{f}{2D}V|V|\right]+\frac{\partial p}{\partial t}+V\frac{\partial p}{\partial s}+c^2\rho\frac{\partial V}{\partial s}=0 \qquad (2.24)$$

Rearranging the terms in above, we have

$$\lambda\left[\frac{\partial V}{\partial t}+\left(V+\frac{c^2\rho}{\lambda}\right)\frac{\partial V}{\partial s}\right]+\left[\frac{\partial p}{\partial t}+\left(V+\frac{\lambda}{\rho}\right)\frac{\partial p}{\partial s}\right]+\lambda\left[g\sin\alpha+\frac{f}{2D}V|V|\right]=0 \qquad (2.25)$$

Assuming,

$$\frac{ds}{dt}=V+\frac{c^2\rho}{\lambda}=V+\frac{\lambda}{\rho} \qquad (2.26)$$

The equality in Eq. 2.26 leads to

$$\lambda=\pm c\rho \qquad (2.27)$$

Which when substituted back into Eq. 2.26 leads to

$$\frac{ds}{dt}=V\pm c \qquad (2.28)$$

Use of proper positive and negative signs of lambda permits writing of the following sets of ordinary differential equations (Wylie, et al.,1993):

$$C^+\begin{cases}\dfrac{dV}{dt}+\dfrac{1}{\rho c}\dfrac{dp}{dt}+g\sin\alpha+\dfrac{fV|V|}{2D}=0\\[2ex]\dfrac{ds}{dt}=V+c\end{cases} \qquad (2.29)$$

and

$$C^- \begin{cases} \frac{dV}{dt} - \frac{1}{\rho c}\frac{dp}{dt} + g\sin\alpha + \frac{fV|V|}{2D} = 0 \\ \frac{ds}{dt} = V - c \end{cases} \tag{2.30}$$

Finally we replace the pressure in favor of total head using (Wylie, et al.,1993) $p = \gamma(H - z)$ and we assume that the entire pipe network is in the same horizontal plane. i.e., $\sin\alpha = 0$.

The new set of characteristic equations can be written as (Wylie, et al.,1993)

$$C^+ : \frac{dV}{dt} + \frac{g}{c}\frac{dH}{dt} + \frac{fV|V|}{2D} = 0 \text{ Only when } \frac{ds}{dt} = V + c \tag{2.31}$$

$$C^- : \frac{dV}{dt} - \frac{g}{c}\frac{dH}{dt} + \frac{fV|V|}{2D} = 0 \text{ Only when } \frac{ds}{dt} = V - c \tag{2.32}$$

2.3.1 Finite Difference Approximation:

We assume that the characteristic curves can be approximated as straight lines over each single Δt interval. But as a result the slopes of C^+ and C^- characteristic lines will not be the same. This can be seen in Fig3 (Larock et al.,2000) as shown below

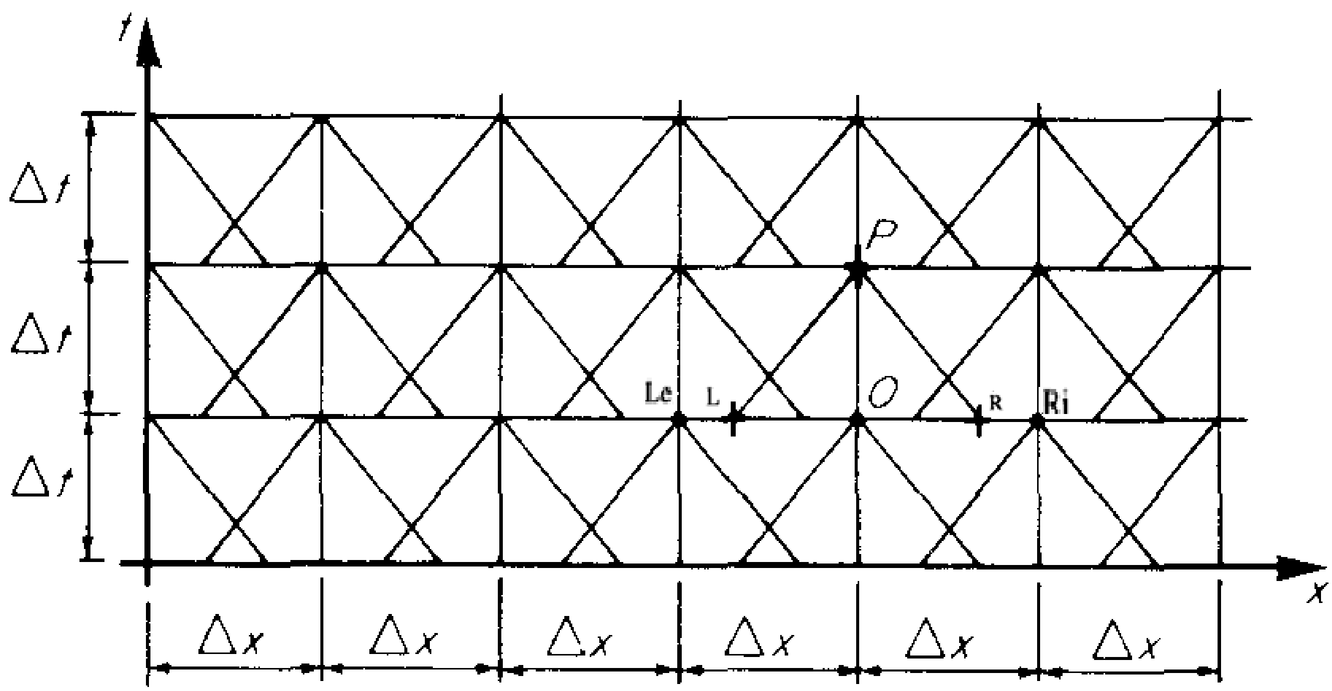


Figure 3. Parameters in the interpolation procedure (Reproduced from Larock et al., 2000)

We can see that the characteristics intersecting at P no longer pass through the grid points L_e and R_i. But instead they pass through the points L and R somewhere between L_e and R_i. Hence, the finite difference approximations to Eqs. 2.31 and 2.32 become

$$\frac{V_P - V_L}{\Delta t} + \frac{g}{c}\frac{H_P - H_L}{\Delta t} + \frac{f}{2D}V_L|V_L| = 0 \quad (2.33)$$

$$\frac{V_P - V_R}{\Delta t} - \frac{g}{c}\frac{H_P - H_R}{\Delta t} + \frac{f}{2D}V_R|V_R| = 0 \quad (2.34)$$

This results in 4 new unknowns $V_{L,} H_L, V_R$ and H_R. But we overcome this problem by choosing Δt so that the point L is near L_e and R is near R_i.. Now linear interpolation becomes an accurate way to evaluate the values of H and V at points L and R.

Along C^+ characteristic (Larock et al., 2000),

$$\frac{\Delta x}{\Delta s} = \frac{V_L - V_C}{V_{Le} - V_C} = \frac{H_L - H_C}{H_{Le} - H_C} \quad (2.35)$$

with

$$\frac{\Delta x}{\Delta t} = \frac{c + V_L}{1} \quad (2.36)$$

Solving above two equations for V_L and H_L yields,

$$V_L = (V_{Le} - V_c)\frac{\Delta x}{\Delta s} + V_C \quad \text{and} \quad H_L = (H_{Le} - H_c)\frac{\Delta x}{\Delta s} + H_c \quad (2.37)$$

Replacing Δx in these equations using the same relation for $\frac{\Delta x}{\Delta t}$ now produces (Larock et al., 2000)

$$V_L = \frac{V_C + c\frac{\Delta t}{\Delta s}(V_{Le} - V_c)}{1 - \frac{\Delta t}{\Delta s}(V_{Le} - V_c)} \quad (2.38)$$

and

$$H_L = H_C + \frac{\Delta t}{\Delta s}(H_{Le} - H_C)(c + V_L) \tag{2.39}$$

Similarly along the C^- characteristic (Larock et al., 2000),

$$V_R = \frac{V_C + c\frac{\Delta t}{\Delta s}(V_{Ri} - V_c)}{1 - \frac{\Delta t}{\Delta s}(V_{Ri} - V_c)} \tag{2.40}$$

And

$$H_R = H_C + \frac{\Delta t}{\Delta s}(H_{Ri} - H_C)(c - V_R) \tag{2.41}$$

Note that $\frac{\Delta t}{\Delta s}(V_{Le} - V_c)$ is on the order of $\frac{V}{c+V}$, which is very small compared to 1, hence the second terms in the denominator of Eqs. 2.38 and 2.40 can be neglected. This would result in

$$V_L = V_C + c\frac{\Delta t}{\Delta s}(V_{Le} - V_c) \tag{2.42}$$

and

$$V_R = V_C + c\frac{\Delta t}{\Delta s}(V_{Ri} - V_c) \tag{2.43}$$

As we now have the known values for $V_{L,} H_L, V_R$ and H_R ,we can solve the Eqs. 2.33 and 2.34 simultaneously for velocity and head at point P.

The solutions are (Larock et al., 2000):

$$V_P = \frac{1}{2}\left[(V_L + V_R) + \frac{g}{c}(H_L - H_R) - \frac{f\Delta t}{2D}(V_L|V_L| + V_R|V_R|)\right] \tag{2.44}$$

$$H_P = \frac{1}{2}\left[\frac{c}{g}(V_L - V_R) + (H_L + H_R) - \frac{c}{g}\frac{f\Delta t}{2D}(V_L|V_L| - V_R|V_R|)\right] \tag{2.45}$$

2.4. Summary of Equations

The two basic equations of fluid flow in closed conduits are a pair of quasilinear partial differential equations (Wylie et al., 1993):

$$\frac{\partial H}{\partial x}+\frac{1}{g}\frac{\partial V}{\partial t}+\frac{\lambda}{2gD}V\left|V\right|=0 \quad \text{(Momentum equation)} \qquad (2.46)$$

$$\frac{\partial H}{\partial t}+\frac{c^2}{g}\frac{\partial V}{\partial x}=0 \quad \text{(Continuity equation)} \qquad (2.47)$$

Using method of characteristics the above pair of partial differential equations can be replaced by an equivalent set (two pairs) of ordinary differential equations which are grouped and identified as C^+ and C^- characteristic (Wylie et al., 1993)

$$C^+\begin{cases}\dfrac{dV}{dt}+\dfrac{g}{c}\dfrac{dH}{dt}+g\sin\alpha+\dfrac{\lambda V\left|V\right|}{2D}=0\\ \dfrac{dx}{dt}=c\end{cases} \qquad (2.48)$$

$$C^-\begin{cases}\dfrac{dV}{dt}-\dfrac{g}{c}\dfrac{dH}{dt}+\dfrac{\lambda V\left|V\right|}{2D}=0\\ \dfrac{dx}{dt}=-c\end{cases} \qquad (2.49)$$

Using finite difference scheme, the above equations can be expressed as (Larock et al., 2000):

$$\begin{aligned}C^+ &: H_P=H_A-B(Q_P-Q_A)-RQ_P\left|Q_A\right|\\ C^- &: H_P=H_B+B(Q_P-Q_B)+RQ_P\left|Q_B\right|\end{aligned} \qquad (2.50)$$

where,

$B=\dfrac{c}{gA}$ And $R=\dfrac{\lambda\Delta x}{2gDA^2}$

The above set of characteristic equations can be further simplified as (Larock et al., 2000):

$$\begin{aligned}C^+ &: H_P=C_P-B_PQ_P\\ C^- &: H_P=C_M+B_MQ_P\end{aligned} \qquad (2.51)$$

Where, $C_P=H_A+BQ_A, B_P=B+RQ_A$ and

(2.52)

$$C_M = H_B - BQ_B, B_M = B + RQ_B$$

C^+ and C^- equations are solved to compute the velocity V_P and the head H_P at each section j. This results in:

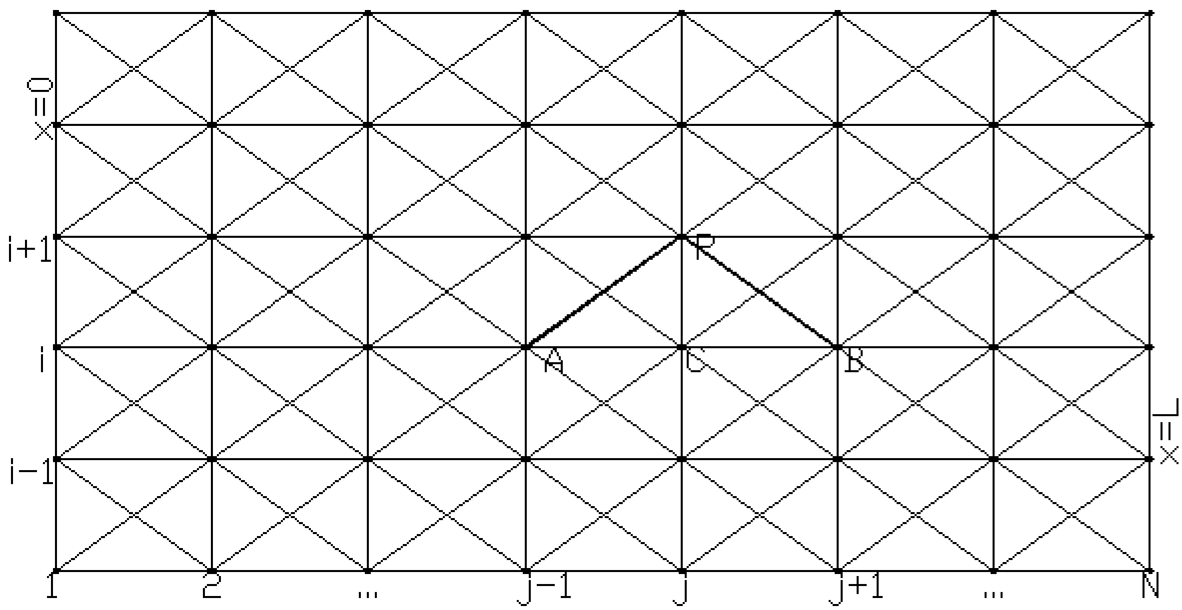


Figure 4. Characterstics shown on a finite difference grid

$$V_{j,i+1} = \frac{1}{2}\left[(V_{j-1,i} + V_{j+1,i}) + \frac{g}{c}(H_{j-1,i} - H_{j+1,i}) - \frac{\lambda \Delta t}{D}(V_{j-1,i}\left|V_{j-1,i}\right| + V_{j+1,i}\left|V_{j+1,i}\right|)\right] \quad (2.53)$$

And

$$H_{j,i+1} = \frac{1}{2}\left[H_{j-1,i} + H_{j+1,i} + \frac{c}{g}(V_{j-1,i} - V_{j+1,i}) - \frac{c}{g}\frac{\lambda \Delta t}{D}(V_{j-1,i}\left|V_{j-1,i}\right| - V_{j+1,i}\left|V_{j+1,i}\right|)\right] \quad (2.54)$$

The basic water hammer conditions for a single pipeline provide the fundamental elements that are necessary for the treatment of more complex piping systems. When the system contains more than one pipeline, the interior sections of each pipeline are treated independently of other parts of the system at each instant of time. The end conditions for each pipeline must interface with adjoining pipelines or with other boundary elements. Again each boundary condition is treated independently of other parts of the system.

2.5 Boundary Conditions

Analyzing the following boundary conditions is important part of water hammer study:

2.5.1 Reservoir

Consider a single pipe system with reservoir at the upstream of the pipe. Hence, a C^- characteristic can be drawn from a node closest to the reservoir towards the junction. Therefore from Eq. 2.51:

$$C^- : H_P = C_M + B_M Q_P \tag{2.55}$$

Here, H_P is the head at the reservoir junction and is assumed constant.

Hence, $H_P = H_R$ and discharge at this junction can be obtained as:

$$Q_P = \frac{H_R - C_M}{B_M} \tag{2.56}$$

2.5.2 Valve

Consider a single pipe system with reservoir upstream and valve at downstream. From the node closest to the valve and towards its left, a C^+ characteristic can be drawn towards the valve junction. Therefore from Eq. 2.51

$$C^+ : H_P = C_P - B_P Q_P . \tag{2.57}$$

During steady state flow through the valve, orifice equation can be applied as (Watters, 1984):

$$Q_P = (C_d A_v)\sqrt{2gH_P} \tag{2.58}$$

Solving above two equations, we obtain

$$Q_P = -gB_P (C_d A_v)^2 + \sqrt{(gB_P (C_d A_v)^2)^2 + 2gC_P (C_d A_v)^2} \quad (2.59)$$

2.5.3 Pumps and Turbines

Pump/Turbine characteristic curves are needed for finding the water hammer caused by them. Let H_S be the head at suction side of the pump. Let H_D be the head at discharge side of the pump. The difference in head (H_D - H_S) gives the total head (H_P) developed by the pump.

$$H_P = f(n, Q_P) \quad (2.60)$$

The above equation (Chaudhry, 1988) is a typical form of pump/turbine characteristic curve. Where, *n* is the pump speed and Q_P is the pump discharge.

Also in this case, the characteristic equations are:

$$\begin{aligned} C^+ &: H_S = C_P - B_P Q_P \\ C^- &: H_D = C_M + B_M Q_P \end{aligned} \quad (2.61)$$

Solving above three basic equations, we obtain discharge and head at the pump during transients.

2.5.4 Junctions

Analyzing network junctions is very important part of transient analysis. Consider a four pipe junction.

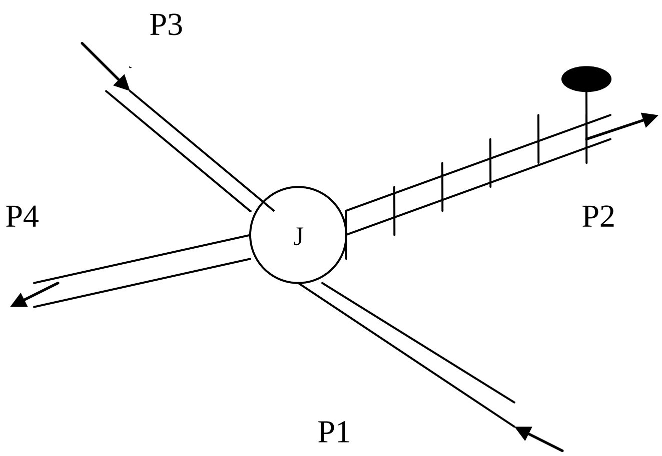


Figure 5. Four pipe junction with valve downstream of one pipe

From figure 5, junction J is the boundary condition for the four pipes. But, if we consider each pipe separately, the junction is present downstream to the pipes P1 and P3, and it is upstream to the pipes P2 and P4 (with respect to the initial flow direction). Hence for pipes where junction is downstream, C^+ line is drawn from pipe nodes towards the junction. Similarly, for pipes where junction is upstream a C^- line is drawn.

The equations (Larock et al., 2000) for the four characteristic lines are:

Pipe 1, C^+ $V_{P1} = C_1 - C_2 H_{P1}$ (2.62)

Pipe 2, C^- $V_{P2} = C_3 + C_4 H_{P2}$ (2.63)

Pipe 3, C^+ $V_{P3} = C_5 - C_6 H_{P3}$ (2.64)

Pipe 4, C^- $V_{P4} = C_7 + C_8 H_{P4}$ (2.65)

In above equations,

$C_2 = \frac{g}{c_1}, C_4 = \frac{g}{c_2}, C_6 = \frac{g}{c_3}, C_8 = \frac{g}{c_4}$ and (2.66)

$$C_1 = (V_{j-1,i} + \frac{g}{c_1} H_{j-1,i}) - \frac{\lambda_1 \Delta t}{2D} V_{j-1,i} |V_{j-1,i}| \quad (2.67)$$

$$C_3 = (V_{j+1,i} - \frac{g}{c_2} H_{j+1,i}) - \frac{\lambda_2 \Delta t}{2D} V_{j+1,i} |V_{j+1,i}|$$

$$C_5 = (V_{j-1,i} + \frac{g}{c_3} H_{j-1,i}) - \frac{\lambda_3 \Delta t}{2D} V_{j-1,i} |V_{j-1,i}|$$

$$C_7 = (V_{j+1,i} + \frac{g}{c_4} H_{j+1,i}) - \frac{\lambda_4 \Delta t}{2D} V_{j+1,i} |V_{j+1,i}|$$

(2.68), (2.69) and (2.70)

Conservation of Mass $V_{P_1} A_1 + V_{P_3} A_3 = V_{P_4} A_4 + V_{P_2} A_2$ (2.71)

Work Energy neglecting head loss at Junction: $H_{P_1} = H_{P_2} = H_{P_3} = H_{P_4}$ (2.72)

Solving these equations for the head values at the junction (Larock et.al, 2000):

$$H_{P_1} = H_{P_2} = H_{P_3} = H_{P_4} = \frac{C_1 A_1 + C_5 A_3 - C_3 A_2 - C_7 A_4}{C_2 A_1 + C_4 A_2 + C_6 A_3 + C_8 A_4} \quad (2.73)$$

Back substitution of these heads into Eqs. 2.62, 2.63, 2.64 and 2.65 yields the velocities in each of the pipes.

Consider a distribution network which is in steady state. Let the valve at the downstream of pipe2 be closed suddenly (as shown in Fig 5) at time t = 0 sec. Each pipe connected to the junction is divided into N number of sections. A rectangular grid can be drawn showing N sections each of length Δs on x-axis and Δt increments on y-axis. At each Δt time step increments head and discharge at each node are calculated using method of characteristics (finite difference approximation).

At the node closest to the valve at upstream end of pipe 2, head and discharge are calculated using valve boundary conditions and at the node closest to the junction, boundary conditions of the junction are used to evaluate head and discharge.

Now consider pipe3, which is an inflow pipe to the Junction:

At the node (of pipe 3) closest to the junction, a negative characteristic from the junction and positive characteristic from the left adjacent node are drawn to meet at the first time step. And subsequently head and discharge are evaluated. So, is the case with all other pipes attached to the junction.

2.6 Transient Flow Analysis

2.6.1 Single pipe system

Consider a single pipe system having reservoir at upstream end (see figure 6) and a valve at the downstream end. The valve is subjected to sudden closure (at time t = 0 sec) and transients in the system is calculated for 2.5 sec.

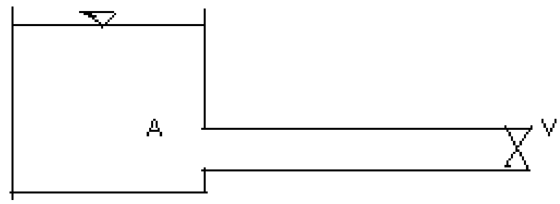


Figure 6. Single pipeline with reservoir upstream and valve downstream

In figure 6, ‘A’ represents the reservoir and ‘V’ represents the valve.

The data required to carry out transient analysis using method of characteristics is shown below.

Length of the pipe = 340 m
Diameter of the pipe = 0.1 m
Darcey friction factor (f) = 0.025
Wave speed in the pipe (a) = 1200 m/s
Valve opening CdA = 0.00015 m^2 (amount of valve opened represented as a factor of pipe area)
Reservoir head = 120 m

Table 1. Transient analysis of single pipe using method of characteristics

	Reservoir	Pipe				Mid Point						Valve	
time. Sec	**QA**	**CM**	**BM**	**Cp**	**Bp**	**CM**	**BM**	**HM**	**QM**	**Cp**	**Bp**	**QV**	**HV**
0	0.0072							**118.197**	0.0072			0.0072	**116.394**
0.1	0.0072	6.576956	15826.45	231.6201	15826.45	4.774048	15826.45	**118.197**	0.0072	229.8172	15826.45	0	**229.817**
0.2	0.0072	6.576956	15826.45	231.6201	15826.45	229.8172	15574.88	**230.711**	0.0001	229.8172	15826.45	0	**229.817**
0.3	-0.0071	229.8172	15576.9	231.6201	15826.45	229.8172	15574.88	**230.711**	0.0001	231.6057	15576.9	0	**231.606**
0.4	-0.0071	229.8172	15576.9	10.19698	15327.41	231.6057	15574.88	**120.015**	-0.0072	231.6057	15576.9	0	**231.606**
0.5	-0.0073	231.6057	15323.38	10.19698	15327.41	231.6057	15574.88	**120.015**	-0.0072	8.423887	15323.38	0	**8.424**
0.6	-0.0073	231.6057	15323.38	6.562525	15319.22	8.423887	15574.88	**7.486**	-0.0001	8.423887	15323.38	0	**8.424**
0.7	0.0072	8.423887	15572.77	6.562525	15319.22	8.423887	15574.88	**7.486**	-0.0001	6.547122	15572.77	0	**6.547**
0.8	0.0072	8.423887	15572.77	231.5913	15826.38	6.547122	15574.88	**118.168**	0.0072	6.547122	15572.77	0	**6.547**
0.9	0.0072	6.547122	15826.45	231.5913	15826.38	6.547122	15574.88	**118.168**	0.0072	229.7888	15826.45	0	**229.789**
1	0.0072	6.547122	15826.45	231.6495	15826.51	229.7888	15574.88	**230.712**	0.0001	229.7888	15826.45	0	**229.789**
1.1	-0.0070	229.7888	15576.96	231.6495	15826.51	229.7888	15574.88	**230.712**	0.0001	231.6346	15576.96	0	**231.635**
1.2	-0.0070	229.7888	15576.96	10.22585	15327.47	231.6346	15574.88	**120.044**	-0.0072	231.6346	15576.96	0	**231.635**
1.3	-0.0073	231.6346	15323.38	10.22585	15327.47	231.6346	15574.88	**120.044**	-0.0072	8.453223	15323.38	0	**8.453**
1.4	-0.0073	231.6346	15323.38	6.533173	15319.15	8.453223	15574.88	**7.485**	-0.0001	8.453223	15323.38	0	**8.453**
1.5	0.0072	8.453223	15572.7	6.533173	15319.15	8.453223	15574.88	**7.485**	-0.0001	6.51728	15572.7	0	**6.517**
1.6	0.0072	8.453223	15572.7	231.5624	15826.32	6.51728	15574.88	**118.139**	0.0072	6.51728	15572.7	0	**6.517**
1.7	0.0072	6.51728	15826.45	231.5624	15826.32	6.51728	15574.88	**118.139**	0.0072	229.7604	15826.45	0	**229.760**
1.8	0.0072	6.51728	15826.45	231.6788	15826.58	229.7604	15574.88	**230.712**	0.0001	229.7604	15826.45	0	**229.760**
1.9	-0.0070	229.7604	15577.02	231.6788	15826.58	229.7604	15574.88	**230.712**	0.0001	231.6635	15577.02	0	**231.663**
2	-0.0070	229.7604	15577.02	10.25471	15327.54	231.6635	15574.88	**120.073**	-0.0072	231.6635	15577.02	0	**231.663**
2.1	-0.0073	231.6635	15323.38	10.25471	15327.54	231.6635	15574.88	**120.073**	-0.0072	8.482552	15323.38	0	**8.483**
2.2	-0.0073	231.6635	15323.38	6.503814	15319.08	8.482552	15574.88	**7.485**	-0.0001	8.482552	15323.38	0	**8.483**
2.3	0.0072	8.482552	15572.63	6.503814	15319.08	8.482552	15574.88	**7.485**	-0.0001	6.48743	15572.63	0	**6.487**
2.4	0.0072	8.482552	15572.63	231.5335	15826.25	6.48743	15574.88	**118.110**	0.0072	6.48743	15572.63	0	**6.487**
2.5	0.0072	6.48743	15826.45	231.5335	15826.25	6.48743	15574.88	**118.110**	0.0072	229.732	15826.45	0	**229.732**

QM, HM represent discharge and the pressure head at the middle of the pipe

QV, HV represent discharge and the pressure head at the upstream of the valve

From above table, the maximum pressure head reached as a result of valve closure = **232m**

Using results given in Table 1, the temporal variation in pressure head during transient condition is shown in figure 7.

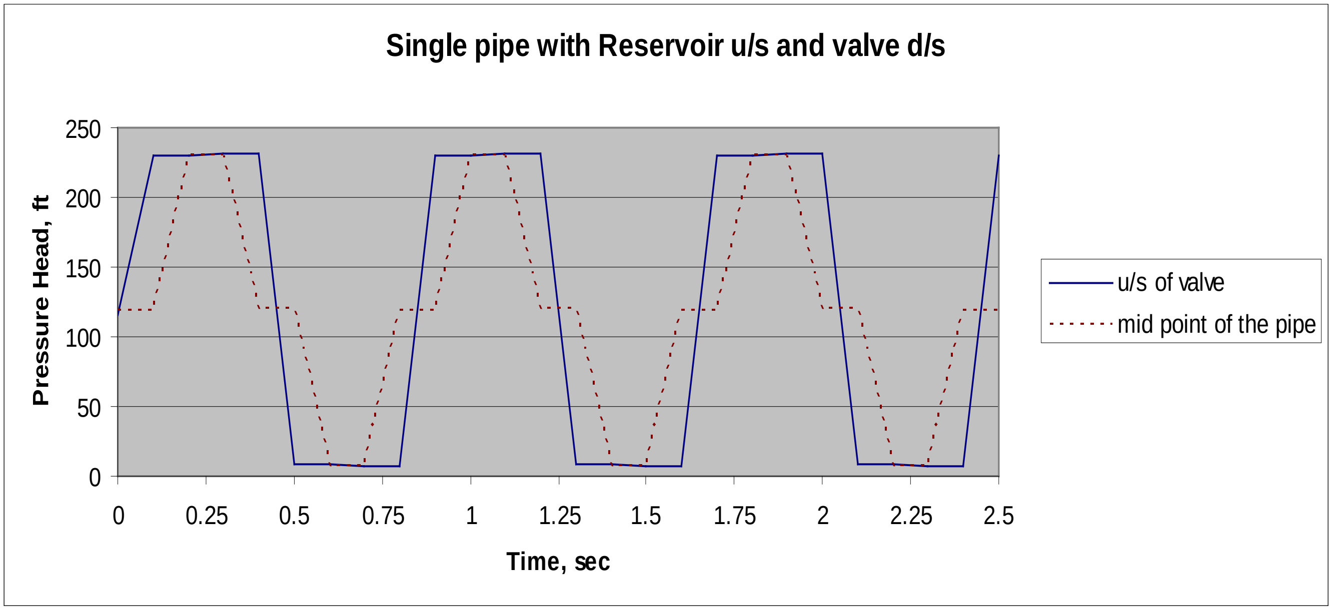


Figure 7. Single pipe with reservoir upstream and valve downstream

From above plots, it is evident that just upstream of the valve (i.e., at the end of the pipe), high pressures are maintained for a longer duration as compared to the middle of the pipe. Hence, pressure surge devices should be placed at pipe joints to avoid failures because they are more susceptible to high pressures.

Analytical solution using Joukowski relation

Joukowski relation (Popescu, 2003) is given by:

$$\Delta H = \pm\left(\frac{a}{g}\right)\Delta V \qquad (2.74)$$

Where,

ΔH = change in pressure head due to valve closure.

a = Wave speed in the pipe.

g = acceleration due to gravity = 9.8 m/s^2

ΔV = change in fluid velocity = 0.91 m/s

Using the same example problem as above, we can find the change in pressure head due to sudden valve closure using Joukowski relation as:

$$\Delta H = \pm \left(\frac{1200}{9.806} \right) 0.91 = 111.4 \text{ m}$$

Steady state pressure head at the valve = Head in the Reservoir = 120 m (neglecting friction losses in the pipe)

Hence, maximum pressure head at the valve = 111.4 m + 120 m = **231 m**

Results

The maximum pressure head at the valve from method of characteristics = 232 m

The maximum pressure head at the valve from Joukowski relation = 231 m

The values obtained from both these analysis are very close. This can be explained by the simplicity of the pipe system we have chosen (single pipe with reservoir at upstream end and valve at downstream end). Simple expressions, such as the Joukowski relation are only applicable under restricted circumstances. The two most important restrictions are:

1) There should be no head loss resulting from friction.
2) There should be no wave reflections (i.e., there is no interaction between devices or boundary conditions in the system).

The above restrictions are narrowly met by the single pipe system discussed above. Hence, the results are comparable.

In above section, transient analysis in a single pipe system is solved using method of characteristics and found that the resulting maximum pressure head value and that obtained by Joukowski relation gave close results.

Most of the times, transient analysis performed on single pipe sytem is useful only for experimental purposes. In practice, most of the distribution systems have multiple pipes and transient flow analysis plays a significant role at pipe joints and at locations where pressure control devices such as valves and pumps exist.

2.6.2 Network Distribution

Consider the network given below (Larock et al., 2000). The Hazen-Williams roughness coefficient is 120 for all pipes. This network experiences a transient that is caused by the sudden closure of a valve at the downstream end of pipe 5. Wave speed is 2850 ft/s for all the pipes. Transient analysis is obtained for this network.

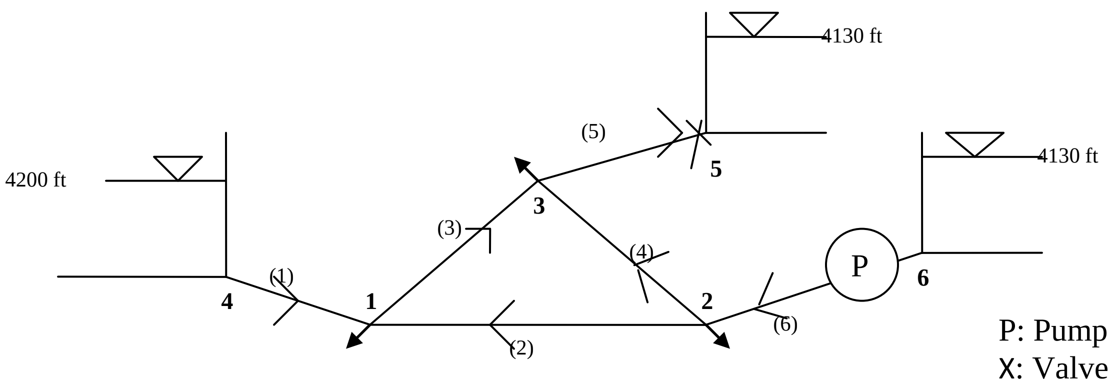


Figure 8. Network distribution system

The pipe information is given in Table 2.

Table 2. Pipe data

Pipe	Length(ft)	Diameter(in)
1	3300	12
2	8200	8
3	3300	8
4	4900	12
5	3300	6
6	2600	14

The node information is given in Table 3.

Table 3. Node data

Node	Elevation(ft)	Demand(gal/min)
1	3800	475
2	3830	317
3	3370	790
4	4050	
5	4000	
6	4010	

Steady state analysis:

First, the above network distribution system is solved for steady state analysis.Following this, transient analysis is performed. A program called "NETWK" was adopted for steady state analysis of above network. "NETWK" is a FORTRAN 95 code (Larock et al., 2000) which can perform steady state analysis on complex pipe systems. The results are shown in Table 4.

Table 4. Pipe data-results of steady flow analysis

Pipe	Discharge(gpm)	Velocity(fps)	Head Loss(ft)
1	340.13	0.96	1.32
2	272.99	1.74	15.7
3	138.11	0.88	1.79
4	1109.95	3.15	17.49
5	-458.06	-5.2	66.89
6	1699.93	3.54	9.64

In Table 4, the negative values for velocity and discharge in pipe 5 show that the actual direction of steady flow is opposite to the initial assumption as shown in figure 8.

Table 5. Node data-results of steady flow analysis

Node	Demand(gpm)	Head(ft)
1	475	398.68
2	317	384.38
3	790	826.89
4	-340.1	150
5	458.1	130
6	-1699.9	214.03

In Table 5, the negative values for demand indicate that the flow direction is into the node. Detailed steady state flow output obtained from "NETWK" program is shown in Appendix A-1. This output is used as one of the input files for the transient analysis.

Transient Analysis:

In order to perform transient analysis, "TRANSNET" program is adopted. "TRANSNET" is the FORTRAN 95 code (Larock et al., 2000) which can perform transient analysis caused by sudden valve closures or pump failures. In pipe network shown in figure 8, the valve located on pipe 5 is closed suddenly at time 0.0 s and below is the summary of results for pipe 5 at time 7.73 s when column separation occurs.

Table 6. Results of transient flow analysis at pipe 5(t =7.73 s)

Pipe	X/L	Head(ft)	Velocity(fps)
5	0	822	-1.1
	0.2	693	-1.14
	0.4	493	-0.56
	0.6	306	-0.35
	0.8	120	-0.27
	1*	-40	0

* at location of the valve

From Table 6, it can be seen that column separation has occurred at location 1 of pipe 5 where pressure head has gone below 0. Detailed transient analysis output table is shown in Appendix A-2. The minimum head attained during transients is -40 ft is already shown in Table 6 and the maximum head is also in pipe 5 at location 0. Its value is 822 ft at time 7.73 sec.

In Appendix B, the author has compared the results of transient analysis with that of WHAMO model. WHAMO (water hammer and model oscillation) model is a hydraulic transient flow model based on an implicit finite difference method and the results where found comparable.

In this chapter, the author has derived the basic equations for solving transient analysis problems using method of characteristics. Two problems are solved. One is a single pipe system which is solved using spreadsheet, and second is a pipe network which is solved using transient analysis program.

In literature most of the transients programs are in FORTRAN language. In chapter 3, the author used Java to code transient analysis program using an object oriented approach. The results of Java program are compared with that obtained from “TRANSNET” program.

CHAPTER 3: An Object Oriented Approach for Transient Analysis in Water Distribution Systems using JAVA programming

3.1 Introduction

Most of the algorithms in computational hydraulics discipline are written in procedural language (FORTRAN, Pascal and C). Procedural programming was found to be adequate for coding moderately extensive programs until 90's (Madan, 2004). In procedural programming, the strategy is based on dividing the computational task into smaller groups termed as functions, procedures or subroutines which perform well-defined operations on their input arguments and have well defined interfaces to other subprograms in the main program.

However, procedural programming approach can get challenging when the code needs to be extended for enhancing the scope of the ptogram. A detailed knowledge of the program is required to work on a small part of the code and poor equivalence between program variables and physical entities further makes it difficult. Integrity of data is another area of concern in procedural programs because, the emphasis is on functions and data is considered secondary. All the functions of a program have access to data and as a result data is highly susceptible to get corrupted when dealing with complex programs. In addition, there are difficulties related to reusability and maintenance of code as procedural programs are platform and version dependent.

Object oriented programming is developed with the objective of addressing some of the typical difficulties associated with procedural programming approach (Madan, 2004). The concept of Object-oriented programming (OOP) began in 1970s and found its first convivial concretization with the Small Talk (Fenves, 1992) language at the beginning of the eighties. Object oriented applications in scientific computing appeared around 1990 and since then its use has been spreading.

Madan (2004) discussed the modeling and design of structural analysis programs using OOP approach. He developed a C++ program for matrix analysis of a space frame. Krishnamoorthy et al., (2002) used object oriented approach to design and develop a genetic algorithm library for solving optimization problems. They later solved a space truss optimization problem by implementing this library. A window-based finite-element analysis program was developed by Ju and Hosain (1996). Graphical finite element objects developed using C++ language were implemented in the program.

Liu et al., (2003) developed an object oriented framework for structural analysis and design. They implemented this framework in optimal design of energy dissipation device (EDD) configurations for best structural performance under earthquake conditions. Tisdale (1996) followed object oriented approach to organize South Florida hydrologic system information as object, dynamic and functional modules. These modules can be used for hydrologic software development.

A real-time flood forecasting system called RIBS (Real time Interactive Basin Simulator) is developed by Garrote and Becchi (1997) using object-oriented framework. Primary applications of RIBS include rainfall forecasting, estimate runoff generating potential of watersheds, and to develop hydrographs. Solomatine (1996) presented a water distribution modeling system (HIS) as an example of object oriented design. He developed two models namely, LinHIS – to solve for linear system of pipes, and NetHIS – to solve for a distributed network of pipes. In both cases, flow is considered either steady or quasi-steady.

Figure 9 depicts the basic difference between the organization of an object –oriented program and a procedure-oriented program. In procedure-oriented program, there are different functions, each of which perform specific action by accessing the data common to all functions whereas in object-oriented program, each object has its own data and methods.

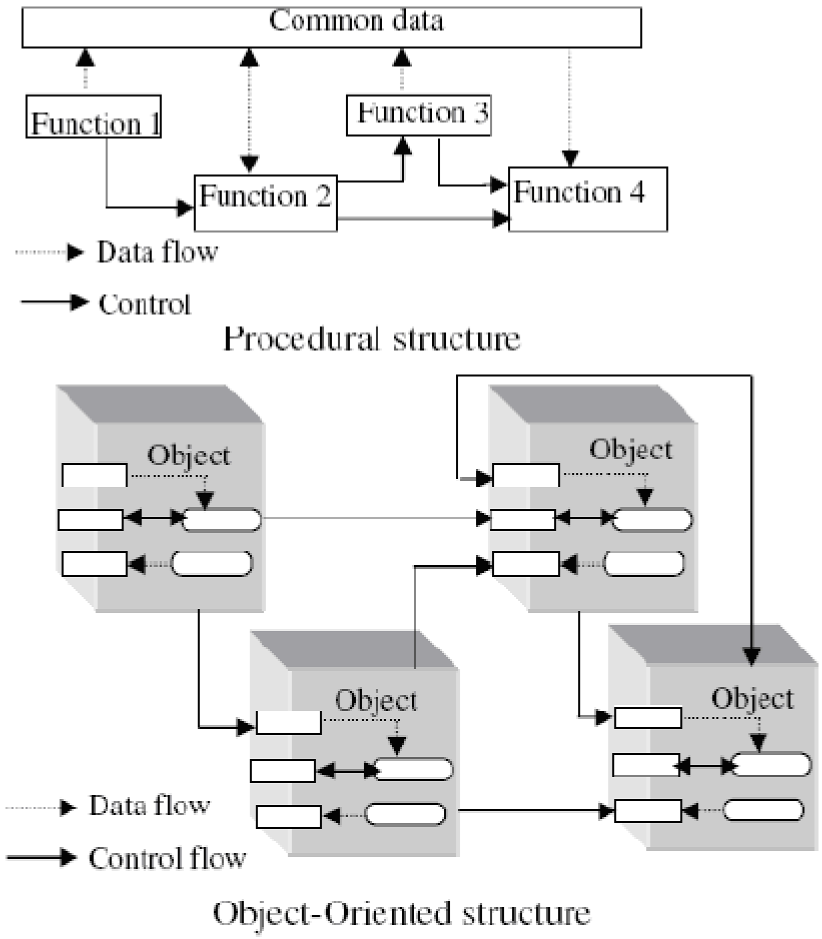


Figure 9. Different types of programming methods (reproduced from www.agiledata.org)

3.2 Object-oriented programming in Java

A Java program describes a community of objects arranged to interact in well-defined ways for a common purpose. None of the objects is sufficient on its own. Each object provides specific services required by other objects in the community to fulfill the program's promise. When an object requires a specific service, it sends a request (called a message) to another object capable of providing that service. The object that receives the message responds by performing actions that often involve additional messages being sent to other objects. This results in a vibrant cascade of messages among a network of objects.

Next the author discusses the use of Java in developing a program for transient analysis in a pipe network. Each of the pipe network elements is represented by a separate class. A detailed description of these classes and primary methods they use are discussed in this chapter. The program can handle pump power failure, suddenly-closing valves, one gradually-closing valve, and sudden demand changes at junctions. A maximum of four pipes can be present at a junction. The program solves for pressure heads and velocity values at any section of the network system after inception of transients. The method of characteristics is used to obtain new pressure head and velocity values from current time step values. The program implements analytical equations discussed in Chapter 1. Figure 10 shows various steps involved in the transient analysis program.

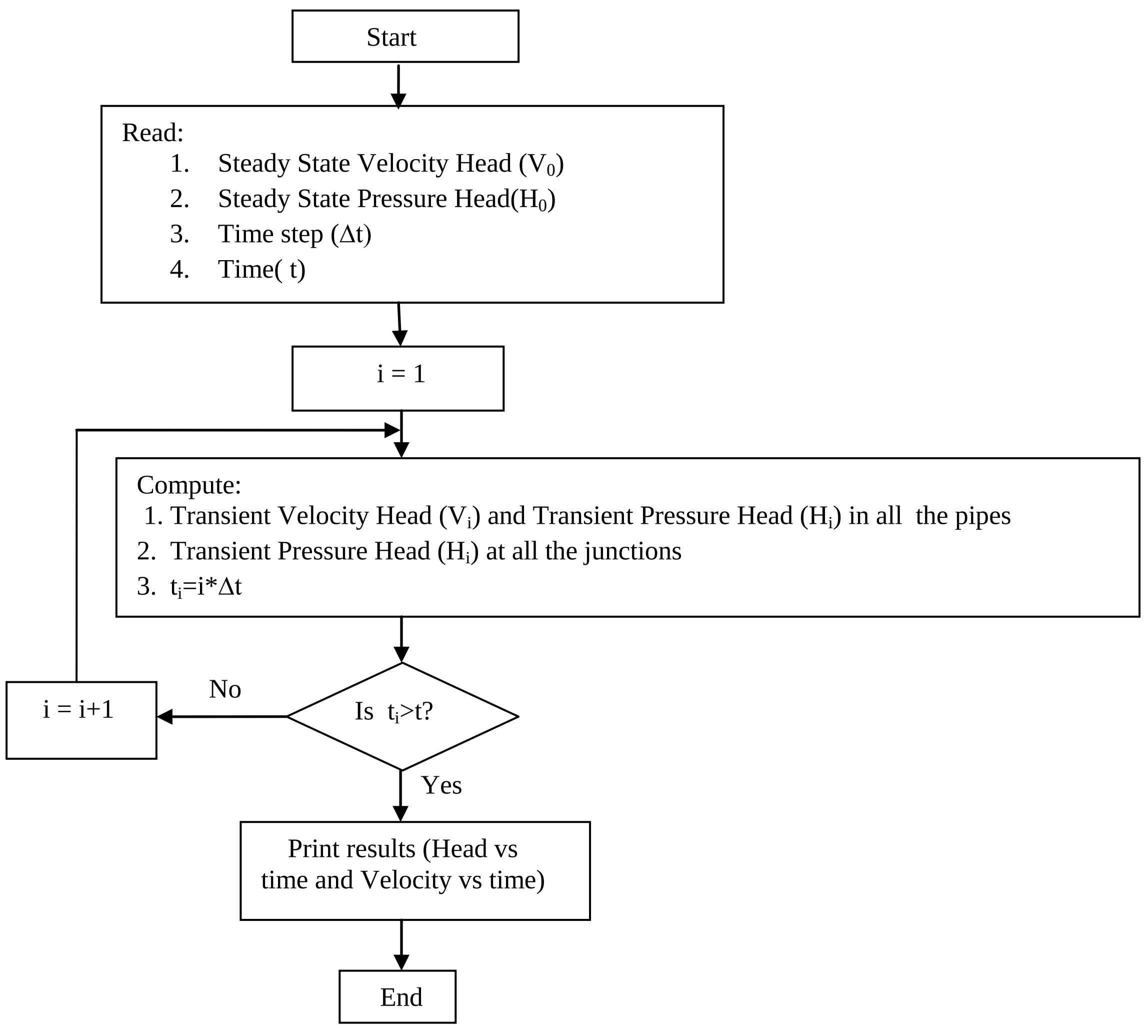


Figure 10. Flow chart to solve for pressure and velocity heads using method of characteristics

The pipe network problem discussed in Section 2.6.2 in Chapter 2 is solved using Java program and results are compared with TRANSNET program output are discussed next.

3.3 Object oriented design

Unified Modeling Language (UML) is widely used tool for object oriented analysis and design. UML diagrams are used to visualize a system by showing interrelationships between classes. Following UML class diagram is used by the JAVA code to solve above discussed pipe network problem:

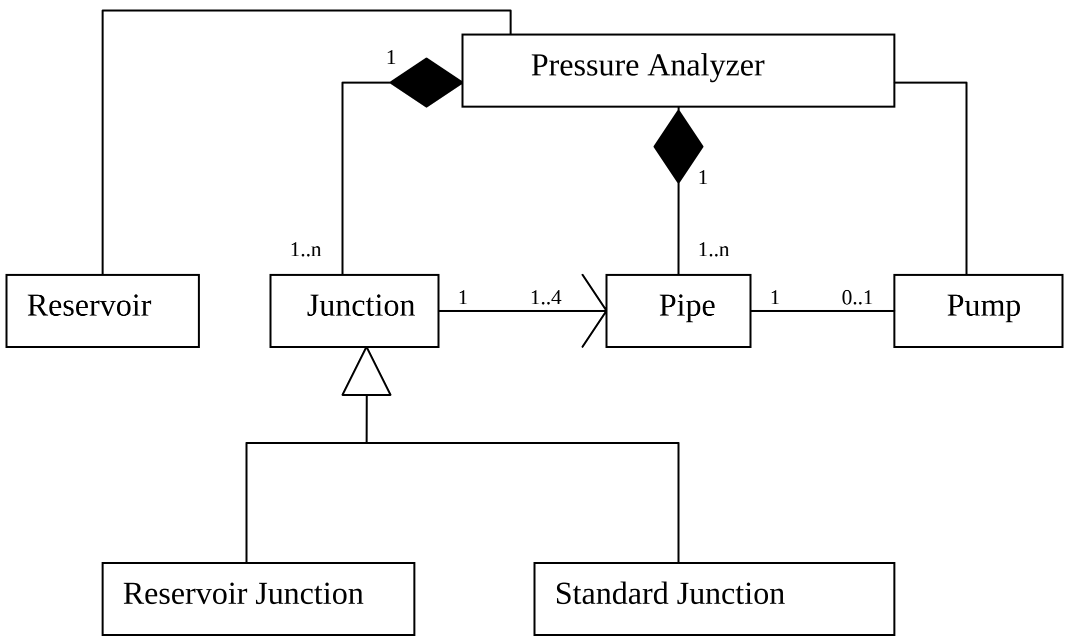
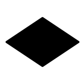


Figure 11. UML class diagram for Transient analysis program



This symbol represents ‘*composition relationship*’ between any two classes. In above class diagram, Pressure Analyzer *is composed of* several junctions, and many pipes. We can also say that pipes and junctions are *part of* Pressure Analyzer class.

This symbol represents ‘*inheritance or dependency relationship*’ between any two classes. In above class diagram, Reservoir junction and Standard Junction are child classes which *inherit* properties from parent class Junction. i.e., Reservoir Junction *is a* type of Junction.

1..4 implies that there can be one or a maximum of four instances of Pipe at each Junction.

0..1 implies that there can be no or one instance of Pump associtated with each Pipe

The program can handle multiple number of pipes and junctions in the network. A maximum of four pipes can meet at a junction. Also, each pipe can have one pump.

The class diagram shown in figure 11 portrays a number of relationships as given in Table 7.

Table 7. Relationships between classes

Class1	Class 2	Relationship
`Reservoir Junction`	`Junction`	Reservoir Junction *"is a"* Junction
`Standard Junction`	`Junction`	Standard Junction *"is a"* Junction
`Junction`	`Pressure Analyzer`	Junction is *"part of"* Pressure Analyzer
`Pipe`	`Pressure Analyzer`	Pipe is *"part of"* Pressure Analyzer

3.4 Classes used in Object oriented program

Following are the classes used in Transient analysis program written in Java. Corresponding methods are listed under each class:

Pipe class

A pipe network may contain more than one pipe. Each of the pipe objects is initialized in the pressure analyzer class. The primary attributes of the pipe class are length, diameter, wavespeed and friction factor. The constructor for the pipe class is shown below.

```
public Pipe(double length, double diameter, double waveSpeed,
                  double frictionFactor) {
            super();
            this.length = length;
            this.diameter = diameter;
            this.waveSpeed = waveSpeed;
            this.frictionFactor = frictionFactor;
      }
```

Some of the methods present in the pipe class are:

Table 8. Methods in Pipe class

Method	**Description**
`computeNextPressureVelocityValues()`	Calculates the pressure and velocity heads at each node of the pipe for the current timestep using the previous timestep values.
`calculateTimePerLength()`	Calculates time step with respect to each pipe which in turn is used to determine time increments for the finite difference scheme.
`calAverageLevel()`	Calculates slope of each pipe based on start and ending node elevations.
`setInitialVelocity(double)`	Updates velocity at each node of the pipe with initial steady state velocity value.

Junction Class

This is an abstract class which only defines the general form of its subclases: Reservoir Junction class and Standard Junction class. The constructor for Junction class is shown below:

```
public Junction(double elevation) {
            super();
            this.elevation = elevation;

      }
```

It also has two abstract methods `getHead()` and `calculateNextHeadVelocity()`, which are overridden in the subclasses.

Pressure Analyser Class

This is main class of the program in which all other classes are initialized and transient anaysis is prefromed. The constructor for Pressure Analyser Class is shown below:

```
PressureAnalyser() {
            doReadMethod();
      }
```

The following methods of this class help to solve transient's problem in any pipe network comprehensively.

Table 9. Methods in Pressure Analyser class

Method	Description
`doReadMethod()`	Reads input data file and updates all the elements of the network such as Pipe, Junction, Reservoir and Pumps.
`Transient_analysis()`	Calculates pressure and velocity heads for each timestep in the network
`doWriteMethod1()`	Writes results to an output text file.
`locateMaxMineadsHVal()`	Locates maximum and minimum heads for each time step

The main( ) method of this class where the program will begin executing is shown below:

```
public static void main(String[] args) {
            PressureAnalyser pan = new PressureAnalyser();
            try {
                  pan.doWriteMethod1();
                  pan.transient_analysis();
                  if (pan.transient_analysis() != 1)
                        pan.doWriteMethod2(); //prints extreme values at
                                                      each pipe node
            }

            catch (Exception ioe) { //Open catch
                  ioe.printStackTrace();
            } // Close catch
            finally { //Open finally
                  pan.closeStreams();
            } // Close finally
      }
```

Pump Class

The java program can handle pumps in pipe network. Typically one pump is associated with each pipe. The following constructor is used in Pressure Analyser class to intialise pump object.

```
public Pump(double speed, double torque, Pipe pipe) {
            super();
            this.speed = speed;
            this.torque = torque;
            this.pipe = pipe;
      }
```

The following methods are present in pump class:

Table 10. Methods in Pump class

Method	Description
`setDHTInfo()`	Reads discharge versus head tables for each pump
`setStage()`	Sets number of stages for each pump

Reservoir Class

Reservoir object is initialized in Pressure Analyser class. Its pressure head is constant throughout the simulation; get and set methods are used to obtain Reservoir head values.

Reservoir Junction Class

This is subclass of Junction Class. Here is the constructor for Reservoir Junction class:

```
public ReservoirJunction(double elevation, Pipe pipe, Reservoir reservoir) {
            super(elevation);
            this.pipe = pipe;
            this.reservoir = reservoir;
      }
```

Methods in Reservoir Junction class are shown below:

Table 11. Methods in Reservoir Junction class

Method	Description
`calculateNextHeadVelocity()`	Computes current Head and Velocity at pipe node closest to the reservoir using previous timestep values.
`Rundown()`	Computes current Head and Velocity at pipe node if a pump is present in the pipeline.
isUpStream( )	Checks if the reservoir object is at upstream or downstream of a pipe

Standard Junction Class:

A junction where two or more pipes meet is termed as Standard Junction. Maximum pipes that can meet at a junction is four. The constructor for the Standard Junction Class is:

```
      public StandardJunction(double elevation, double demand, ArrayList
pipes) {
            super(elevation);
            this.demand = demand;
            this.pipes = pipes;
```

```
}
```

Following are the methods in Standard Junction Class:

Table 12. Methods in Standard Junction class

Method	**Description**
`calculateNextHeadVelocity()`	Computes current Head and Velocity at pipe node closest to the Junction using previous timestep values.
isUpStream( )	Checks if the pipe object is at upstream or downstream of the Junction

3.5 Comparison between TRANSNET and JAVA

TRANSNET is the Fortran code for transient analysis. The following portion of TRANSNET code computes Pressure and Velocity heads for current time step at upstream end (for any of reservoir, demand or pump boundary conditions) using previous time step values.

```
 80  I=IABS(NODEP(III,1))
     VRITE=V(I,1)-RATIO(I)*A(I)*(V(I,1)-V(I,2))
     HRITE=H(I,1)-RATIO(I)*A(I)*(H(I,1)-H(I,2))
     IF(NTYPE(III).EQ.99) GO TO 85
     CC=VRITE-C(I)*HRITE-C(I)*DELT*VRITE*SINE(I)-AK(I)*VRITE*ABS(VRITE)
     IF(NSHUT.EQ.0) GO TO 800    //check for suddenly closed valves //
     II=NODEP(III,1)
     DO 801 N=1,NSHUT
     IF(ISHUT(N).NE.II) GO TO 801
     VNEW(I,1)=0.
     HNEW(I,1)=-CC/C(I)
     HNODE(III)=HNEW(I,1)-ELG(III) //calculates new Head at each node //
     ELHGL(III)=HNEW(I,1)
     GO TO 40
 801 CONTINUE
 800 CONTINUE
     IF(ABS(QNEW(III)).LE..00001) GO TO 802
     VNEW(I,1)=QDEM(III)/AREA(I) //calculates new velocity at each node //
     HNEW(I,1)=(VNEW(I,1)-CC)/C(I)
     ELHGL(III)=HNEW(I,1)
```

```
      GO TO 40
  802 HNEW(I,1)=ELHGL(III)
      VNEW(I,1)=CC+C(I)*HNEW(I,1)
      GO TO 40
```

The corresponding code in Java for Transient analysis program from "*ReservoirJunction*" class is shown below

```
Pump pump = pipe.getPump();

                  pipeJuncVel = pipe.getVelocityAtParts()[0];
                  pipeNextPrevVel = pipe.getVelocityAtParts()[1];
                  pipeJuncPress = pipe.getPressureAtParts()[0];
                  this.head= pipe.getPressureAtParts()[0];
                  pipeNextPrevPress = pipe.getPressureAtParts()[1];
                  VRITE = pipeJuncVel - timeIntPerLen * pipeWaveSpeed
                              * (pipeJuncVel - pipeNextPrevVel);
                  HRITE = pipeJuncPress - timeIntPerLen * pipeWaveSpeed
                              * (pipeJuncPress - pipeNextPrevPress);
            double CC;


                  if(pump == null){

                         CC = VRITE - inverseSpeed * HRITE - inverseSpeed
                              * timeInterval * VRITE * avgLevel -
                              frictionRatio * VRITE* Math.abs(VRITE);

                  if (pipe.getSide() > 0 ) {

                        pipe.getNewVelocityAtParts()[0] = 0.0;
                        pipe.getNewPressureAtParts()[0] = -CC /inverseSpeed;
                              setHeadAtNode( pipe.getNewPressureAtParts()[0]
                              - getElevation());
                        head = pipe.getNewPressureAtParts()[0];
                        return;
                  }

                  if (Math.abs(getNewDemand()) > .00001) {
                        pipe.getNewVelocityAtParts()[0] = demand /
                             pipe.getArea();
                        pipe.getNewPressureAtParts()[0] =
                        (pipe.getVelocityAtParts()[0] - CC)/inverseSpeed;
                        head = pipe.getNewPressureAtParts()[0];
                        return;
                  }
                  pipe.getNewPressureAtParts()[0] = head;
                  pipe.getNewVelocityAtParts()[0] = CC + inverseSpeed
                              * pipe.getNewPressureAtParts()[0];
                  return;
                  }
```

Table 13 compares various features of TRANSNET and Transient analysis program.

Table 13. Comparative advantages of the object- oriented programming

TRANSNET	Transient analysis program
1) The variables program uses does not relate to the pipe network attributes. Hence, a secondary reference is necessary to understand the code. e.g., VNEW, HNEW.	1) Direct correlation exists between the variables and the pipe network elements. e.g., getNewVelocityAtParts, getNewPressureAtParts.
2) *"go to"* is the control statement used very often to cause the flow of program execution to advance and to make changes to the state of the program. This makes the code hard to comprehend as the reader needs to juggle between different parts of the program	2) *"go to"* is very rarely used in Java. *"if"* is the control statement used. All the code which needs to be executed is specified in a single block.
3) Expansion of code is very difficult as developer needs to make changes at various portions of code.	3) This is very flexible. Adding new methods and new classes is possible without modifying the entire program.

3.6 Test problem and verification of results

The pipe network problem described in Section 2.6.2 in Chapter 2 is solved using the Java program. The input and output data for this program are in text file format, which are shown in Appendices C-1 and C-2 respectively.

The results obtained from Transient analysis program are compared with TRANSNET program, which are shown in Table 14.

Table 14. Maximum pressure head (in feet) values during transients in pipe network

Pipe	Node Location*	Result from Transient analysis	Results from TRANSNET	Percent error
Pipe 1	**0.6**	359.8	359.7	**0.028**
Pipe 2	**0.583**	538.2	538.2	**0.000**
Pipe 3	**0.4**	696.9	697.2	**0.043**
Pipe 4	**0.286**	635.7	635.8	**0.016**
Pipe 5	**0.8**	761.7	762.4	**0.092**
Pipe 6	**0.75**	431.8	431.6	**0.046**

*represents distance in terms of pipe length from upstream end of pipe

The results from Transient analysis program and TRANSNET program are quite comparable. However, the slight difference in values between these two programs is due to the rounding error. All the values calculated in Java are third decimal accurate where as in TRANSNET computations are done until first decimal only.

Chapter 4: Modeling Transient Gaseous Cavitation in Pipes

4.1 Introduction

During transient flow of liquid in pipelines, pressures sufficiently less than the saturation pressure of dissolved gas can be reached. As a result, gas bubbles are formed due to diffusion of cavitation nuclei. In this process of gas release, free gas volume increases. Consequently, the mixture celerity is reduced due to added compressibility of the gas, which in turn may give rise to significant pressure wave dispersion.

The decision to account for gas release (Wiggert and Sundquist, 1979) during a pressure transient in a pipe depends upon the system dimensions, type of fluid mixture being transported, extent of saturation of the gas, and low pressure residence times. In long pipelines where big elevation difference exists at different sections of pipe, transient pressures below gas saturation pressure is possible. Also, in highly soluble solutions such as water and carbon dioxide or hydraulic oil and air, significant gas release can take place and should be considered to correctly simulate the transient. Examples of gaseous cavitation are found in large-scale cooling water units, aviation fuel lines, and hydraulic control systems.

In present work, a two dimensional (2D) numerical model is developed to analyze gaseous cavitation in a single pipe system. The model is based on mathematical formulations proposed by Cannizzaro and Pezzinga (2005) and Pezzinga (2003). The model considers energy dissipation due to both thermic exhange between gas bubbles and surrounding liquid and during the process of gas release where as Cannizzaro and Pezzinga (2005) and Pezzinga (2003) present separate models for thermic exchange and gas release.

The model considers liquid flow with a small amount of free gas. Gas release expression is derived from Henry's law. The energy expression in complete form is from Anderson (1995) and Saurel and Le Metayer (2001). The thermic exchange is expressed from Newton's law of cooling (Ewing, 1980). These are the basic equations model uses which undergo several transformations. Finally, four constitutive equations – rate of mass increase of free gas, the mixture continuity equation, mixture energy equation, and the mixture momentum – yield a set of differential equations that are solved by the MacCormack (1969) explicit finite-difference method.

Later, an example is presented to demonstrate this model to analyse transients for the closure of a downstream valve in a pipe containing an air-water mixture with reservoir upstream.

4.2 Free Gas in Liquids

The model assumes free gas to be distributed throughout the pipe as a homogeneous bubbly-fluid mixture with gas bubbles and liquid moving at same velocity. A void fraction α , is used to describe the ratio of volume of free gas, V_g , to the mixture volume, *V*, and for a given mass of free gas, it is pressure dependant (Wylie, 1984).

$$\alpha = \frac{V_g}{V} \tag{4.1}$$

Dalton's law states that the total pressure exerted by a mixture of gases is equal to sum of partial pressures of various components. If air were the free gas distributed in water in a pipeline the bubbles would contain a mixture of air and water vapor. The total volume occupied by air would be the same volume as occupied by the water vapor. Dalton's law is expressed (Wylie, 1984):

$$P^* = P_g^* + P_v^* \tag{4.2}$$

in which P^* is the total absolute pressure, P_g^* is the absolute partial pressure of air and P_v^* is the absolute vapor pressure.

For small void fractions isothermal behavior of free gas is a valid assumption. With a given mass of free gas, M_g , and use of the perfect gas law (Wylie, 1984):

$$P_g^* V_g = M_g R_g T \tag{4.3}$$

and $P_v^* V_v = M_v R_v T$ (4.4)

In which V_g is the gas volume, R_g is the gas constant, T is the absolute temperature, V_v is the vapor volume and is equal to the free gas volume, M_v is mass of vapor and R_v is the vapor gas constant.

We define mass of free gas per unit volume of the mixture as:

$$m = \frac{M_g}{V} \tag{4.5}$$

But from above, $M_g = P_g^* V_g / R_g T$. Therefore, m can be rewritten as:

$$m = \frac{P_g^* V_g}{R_g T V} \tag{4.6}$$

Hence, from above:

$$\alpha = \frac{V_g}{V} = \frac{m R_g T}{P_g^*} \tag{4.7}$$

From (4.5) and (4.7)

$$\frac{M_g}{V_g} = \rho_g = \frac{m}{\alpha} \tag{4.8}$$

4.3 Rate of Gas Release

During low pressure periods, gas release takes place and should be considered to correctly simulate the transient. The present model accommodates for gas release using Henry's law.

***Henry's law*:** The amount of gas dissolved in a liquid is proportional to the partial pressure of gas above the liquid(www).

$$P = K_H C_W \tag{4.9}$$

Where K_H is the Henry's law constant, P is the partial pressure of gas and C_w is concentration of dissolved gas in mol / L.

The dimensionless Henry's law constant, $K_H^{'}$ is defined as

$$K_H^{'} = \frac{C_a}{C_w} \quad (4.10)$$

C_a is the concentration in gas phase. Using $PV = nRT$ and $\frac{n}{V} = C_a$ we have

$$C_a = \frac{P}{RT} \quad (4.11)$$

From Eqs. 4.9, 4.10 and 4.11 we have

$$K_H^{'} = \frac{K_H}{RT} \quad (4.12)$$

Mass of dissolved gas in aqueous phase, $m_{aq} = C_w V_l M$

$$m_{aq} = C_w V_l M = \frac{P}{K_H} V_l M = \frac{P}{K_H^{'} RT} V_l M \quad (4.13)$$

The increase in mass of free gas $= m_{aq,s} - m_{aq}$

$$m_{aq,s} - m_{aq} = \frac{V_l M}{K_H^{'} RT} [p_s - p] \quad (4.14)$$

Where $m_{aq,s}$ is the initial mass of free gas present at equilibrium, and p_s is the saturation or equilibrium pressure. Hence, the rate of increase in mass of free gas is given by:

$$\frac{\partial m}{\partial t} = \frac{m_{aq,s} - m_{aq}}{\Delta t} = \frac{V_l M}{\Delta t} \frac{1}{K_H^{'} RT} [p_s - p] \quad (4.15)$$

4.4 Energy Equation of gas as function of Temperature and Pressure

The energy equation for a gas of unit mass is written as (Anderson, 1995):

$$dq - pdV = dU \quad (4.16)$$

Where dq is the heat added in Joules, $pdV = dW$ is the work done by the gas in Joules, dU is the change in internal energy in Joules.

$$\frac{dq}{dt} = \frac{dU}{dt} + p\frac{dV}{dt} \quad (4.17)$$

From the definition of enthalpy (Anderson, 1995)

$$h = U + pV \tag{4.18}$$

Where h is the enthalpy per unit mass.

$$\frac{dh}{dt} = \frac{dU}{dt} + p\frac{dV}{dt} + V\frac{dp}{dt} \tag{4.19}$$

As we are considering unit mass of gas, $V = 1/\rho_g$ where, ρ_g is the density of gas.

Hence, above equation can be written as:

$$\frac{dh}{dt} = \frac{dU}{dt} + p\frac{dV}{dt} + \frac{1}{\rho_g}\frac{dp}{dt} \tag{4.20}$$

From (4.17) and (4.20)

$$\frac{dh}{dt} = \frac{dq}{dt} + \frac{1}{\rho_g}\frac{dp}{dt} \tag{4.21}$$

The above is another form of expressing energy equation of a gas.

From the definition of specific heat at constant pressure, heat Q added to raise the temperature of a mass of *m* from T_1 to T is (Saurel and Le Metayer, 2001):

$$Q = mc_p(T - T_1) \tag{4.22}$$

For unit mass: $q = c_p(T - T_1)$ (4.23)

$$\frac{dq}{dt} = c_p\frac{dT}{dt} \tag{4.24}$$

4.4.1 Relation between c_p and c_v

The energy equation for a reversible process of a closed system, ideal gas is (Anderson, 1995):

$$dq = dU + pdV \tag{4.25}$$

Where q is the heat transfer per unit mass, U is the internal energy per unit mass.

Ideal gas equation for unit mass of gas can be written as (Anderson, 1995):

$$pV = RT \quad (4.26)$$

At constant pressure, $pdV = RdT$ (4.27)

From above:

$$dq = dU + RdT \quad (4.28)$$

$$\frac{dq}{dT} = \frac{dU}{dT} + R \quad (4.29)$$

Specific heat(www) of a gas is defined as the amount of heat required to change unit mass of gas by one degree in temperature.

Hence, specific heat of gas at constant volume c_v is $\frac{dq}{dT}$ when $dV = 0$. From energy equation

$$dq = dU \quad (4.30)$$

$$\frac{dq}{dT} = \frac{dU}{dT} = c_v \quad (4.31)$$

$$d(pV) = pdV + Vdp \quad (4.32)$$

$$pdV = d(pV) - Vdp \quad (4.33)$$

Hence, the above energy equation can be rewritten as:

$$dq = dU + d(pV) - Vdp \quad (4.34)$$

$$dq = d(U + pV) - Vdp \quad (4.35)$$

From the definition of enthalpy, H (Anderson, 1995)

$$H = U + pV \quad (4.36)$$

Therefore at constant pressure, the energy equation is:

$$dq = dH \quad (4.37)$$

Specific heat of gas at constant pressure c_p:

$$\frac{dq}{dT} = \frac{dh}{dT} = c_p \tag{4.38}$$

Hence from Eq.s 4.29, 4.31 and 4.38

$$c_p = c_v + R \tag{4.39}$$

Let k be the ratio of specific heats,

$$k = \frac{c_p}{c_v} \tag{4.40}$$

From Eq.s 4.39 and 4.40

$$c_p(1 - \frac{1}{k}) = R \tag{4.41}$$

$$c_p = \frac{k}{(k-1)} R \tag{4.42}$$

4.4.2 Newton's law of cooling

Newton's Law of Cooling states that the rate of change of temperature of an object is proportional to the difference between its own temperature and the ambient temperature (Ewing, 1980).

$$\frac{dT}{dt} = -K_N (T - T_0) \tag{4.43}$$

Where T is the absolute temperature of the body (gas), T_0 is the absolute temperature of the surrounding (liquid), K_N is the experimental positive constant (1/sec). Hence, from above:

$$\frac{dq}{dt} = -c_p K_N (T - T_0) \tag{4.44}$$

From Eq. 4.38, $\frac{dh}{dt} = c_p \frac{dT}{dt}$ (4.45)

Hence, energy Eq. 4.21 can be written as:

$$c_p \frac{dT}{dt} = -c_p K_N (T - T_0) + \frac{1}{\rho_g} \frac{dp}{dt} \tag{4.46}$$

Using equation (4.42) in above along with the gas law for ρ_g , we have

$$\frac{dT}{dt} - \frac{(k-1)}{k}\frac{T}{p}\frac{dp}{dt} + K_N(T - T_0) = 0 \tag{4.47}$$

4.5 One-Dimensional Two-Phase Flow

Present model considers the fluid as a homogeneous two-phase air-water mixture. The properties of the fluid will be average of both components. Following are the assumptions used in the model:

1) Gas bubbles are distributed throughout the pipe and they are very small compared to pipe diameter.
2) Difference in pressure due to surface tension across a bubble surface can be neglected; and
3) Momentum exchange between gas bubbles and surrounding liquid is negligible, so that gas bubbles and liquid have the same velocity.

4.5.1 Mixture Density

The density of a liquid gas mixture, ρ can be expressed as (Wiggert and Sundquist, 1979)

$$\rho = \alpha\rho_g + (1-\alpha)\rho_l \tag{4.48}$$

Where ρ_l is the liquid density. Using Eq. 4.8, above can be written as:

$$\rho = m + (1-\alpha)\rho_l \tag{4.49}$$

$$\frac{\partial\rho}{\partial t} = \frac{\partial m}{\partial t} + \frac{\partial\rho_l(1-\alpha)}{\partial t} - \rho_l\frac{\partial\alpha}{\partial t} \tag{4.50}$$

Bulk modulus of elasticity of a liquid is given by (Larock, 2000):

$$E = -\frac{dp}{dv/v} = \frac{dp}{d\rho_l/\rho_l} \tag{4.51}$$

Also $E/\rho_l = c^2$ (4.52)

Hence, we can write:

$$\frac{\partial \rho_l}{\partial t} = \frac{\rho_l}{E}\frac{\partial p}{\partial t} = \frac{1}{c^2}\frac{\partial p}{\partial t} \qquad (4.53)$$

Using Eq. 4.53 in Eq. 4.50 we have

$$\frac{\partial \rho}{\partial t} = \frac{1}{c^2}\frac{\partial p}{\partial t}(1-\alpha) - \rho_l\left[-\frac{mRT}{p^2}\frac{\partial p}{\partial t} + \frac{mR}{p}\frac{\partial T}{\partial t} + \frac{RT}{p}\frac{\partial m}{\partial t}\right] + \frac{\partial m}{\partial t} \qquad (4.54)$$

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} = \frac{\partial p}{\partial t}\left[\frac{1}{\rho c^2}(1-\alpha) + \frac{\rho_l}{\rho}\frac{mRT}{p^2}\right] - \frac{\partial T}{\partial t}\left(\frac{mR\rho_l}{p\rho}\right) + \frac{\partial m}{\partial t}\left[\frac{1}{\rho} - \frac{RT}{p}\frac{\rho_l}{\rho}\right] \qquad (4.55)$$

For $\alpha << 1$ and $\rho_l \approx \rho$ we have

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} = \frac{\partial p}{\partial t}\left[\frac{1}{\rho c^2}(1-\alpha) + \frac{\rho_l}{\rho}\frac{mRT}{p^2}\right] - \frac{\partial T}{\partial t}\left(\frac{mR\rho_l}{p\rho}\right) + \frac{\partial m}{\partial t}\left[\frac{1}{\rho} - \frac{RT}{p}\frac{\rho_l}{\rho}\right] \qquad (4.56)$$

4.5.2 Continuity Equation

From one dimensional continuity equation given in Wylie et al.,(1993)

$$\frac{1}{\rho}\frac{d\rho}{dt} + \frac{1}{A}\frac{dA}{dt} + \frac{\partial u}{\partial x} = 0 \qquad (4.57)$$

The second term in above equation deals with elasticity of the pipe and its rate of deformation with pressure. Assuming pipe is rigid $\frac{dA}{dt}$ can be scaled to zero. As a consequence, we have constant area and

$$\frac{\partial u}{\partial x} = \frac{1}{A}\frac{\partial Q}{\partial x}\text{. Where } Q = AV \qquad (4.58)$$

Using above expression in continuity equation Eq. 4.57 we have:

$$\frac{1}{\rho}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x}\right] + \frac{1}{A}\frac{\partial Q}{\partial x} = 0 \qquad (4.59)$$

Also neglecting the spatial derivative of density in above continuity equation we have,

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t}+\frac{1}{A}\frac{\partial Q}{\partial x}=0 \tag{4.60}$$

From Eqs. 4.56 and 4.60 continuity equation is finally expressed as:

$$\frac{\partial p}{\partial t}\left[1+\frac{\rho c^2 mRT}{p^2}\right]+\frac{\partial m}{\partial t}\left[c^2-\frac{\rho c^2 RT}{p}\right]-\frac{\partial T}{\partial t}\left[\frac{\rho c^2 mR}{p}\right]+\frac{\rho c^2}{A}\frac{\partial Q}{\partial x}=0 \tag{4.61}$$

4.6 Conservation form of Mixture Continuity and Energy Equations

$$Define\ \phi=\frac{p}{\rho g}-\frac{c^2 mRT}{pg}+\frac{mc^2}{\rho g} \tag{4.62}$$

$$\frac{\partial \phi}{\partial t}=\frac{1}{\rho g}\frac{\partial p}{\partial t}-\frac{1}{\rho^2}\frac{p}{g}\frac{\partial \rho}{\partial t}+\frac{c^2}{p^2}\left(\frac{mRT}{g}\right)\frac{\partial p}{\partial t}-\frac{c^2 RT}{pg}\frac{\partial m}{\partial t}-\frac{c^2 mR}{pg}\frac{\partial T}{\partial t}-\frac{mc^2}{\rho^2 g}\frac{\partial \rho}{\partial t}+\frac{c^2}{\rho g}\frac{\partial m}{\partial t} \tag{4.63}$$

Assuming $\frac{\partial \rho}{\partial t}\approx 0$

$$\frac{\partial \phi}{\partial t}=\frac{\partial p}{\partial t}\left[\frac{1}{\rho g}+\frac{c^2}{g}\frac{mRT}{p^2}\right]+\frac{\partial m}{\partial t}\left[\frac{c^2}{\rho g}-\frac{c^2}{g}\frac{RT}{p}\right]-\frac{\partial T}{\partial t}\left[\frac{c^2}{g}\frac{mR}{p}\right] \tag{4.64}$$

Using above, mixture continuity equation, Eq. 4.61 is expressed as:

$$\rho g\frac{\partial \phi}{\partial t}+\frac{\rho c^2}{A}\frac{\partial Q}{\partial x}=0 \tag{4.65}$$

Finally, the following conservation form of mixture continuity equation is obtained (Cannizzaro and Pezzinga, 2005):

$$\frac{\partial \phi}{\partial t}+\frac{c^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{4.66}$$

Define gas entropy $s=c_p\left[\ln T-\frac{k-1}{k}\ln p\right]$ (4.67)

$$\frac{\partial s}{\partial t} = c_p \left[\frac{1}{T}\frac{\partial T}{\partial t} - \frac{k-1}{k}\frac{1}{p}\frac{\partial p}{\partial t} \right] \tag{4.68}$$

From energy equation, Eq. 4.47, which is:

$$\frac{dT}{dt} - \frac{(k-1)}{k}\frac{T}{p}\frac{dp}{dt} + K_N(T - T_0) = 0$$

$$T\left[\frac{1}{T}\frac{dT}{dt} - \frac{(k-1)}{k}\frac{1}{p}\frac{dp}{dt} \right] + K_N(T - T_0) = 0 \tag{4.69}$$

From Eq.s 4.68 and 4.69 we have

$$T\left[\frac{1}{c_p}\frac{\partial s}{\partial t} \right] + K_N(T - T_0) = 0 \tag{4.70}$$

$$\frac{\partial s}{\partial t} + c_p K_N \left(1 - \frac{T_0}{T}\right) = 0 \tag{4.71}$$

The conservation form of Energy equation is (Cannizzaro and Pezzinga, 2005):

$$\frac{\partial s}{\partial t} = c_p K_N \left(\frac{T_0}{T} - 1\right) \tag{4.72}$$

4.7 Mixture Momentum Equation

The following momentum equations are written for an elastic pipe with circular cross section, and assume a 2D flow field with axial symmetry (Pezzinga, 1999).

Along longitudinal direction, x-axis momentum equation:

$$\rho\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} \right] = -\frac{\partial p}{\partial x} - \frac{\partial \sigma_x}{\partial x} - \frac{1}{r}\frac{\partial (r\tau)}{\partial r} \tag{4.73}$$

Along radial direction, r-component momentum equation:

$$\rho\left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial r} \right] = -\frac{\partial p}{\partial r} - \frac{\partial \tau}{\partial x} - \frac{1}{r}\frac{\partial (r\sigma_r)}{\partial r} + \frac{\sigma_\theta}{r} \tag{4.74}$$

For laminar and turbulent unsteady flows, radial velocity component has values of order $10-20\mu m/s$. Hence, in above momentum equations radial velocity v and its derivatives are neglected. Also, in the momentum equation, in the longitudinal direction the normal stress is assumed to be equal to the pressure and residual convection term is neglected as usually done also in 1D models.

The x-axis momentum equation can be rewritten as:

$$\rho\frac{\partial u}{\partial t}+\frac{\partial p}{\partial x}+\frac{1}{r}\frac{\partial(r\tau)}{\partial r}=0 \tag{4.75}$$

Neglecting the stress terms in the radial momentum equation can be written as:

$$\frac{\partial p}{\partial r}=0 \tag{4.76}$$

4.7.1 Stress Model

Selection of appropriate stress model is important to estimate the initial velocity profile. In the present gaseous cavitation model for turbulent flow, a two-zone model is considered. In the viscous sub layer Newton's law is used and in turbulent zone, Prandtl mixing length hypothesis (Pezzinga, 1999), namely:

$$\tau=-\rho\nu\frac{\partial u}{\partial r}-\rho l^{2}\left|\frac{\partial u}{\partial r}\right|\frac{\partial u}{\partial r} \tag{4.77}$$

is used. In above, ν = kinematic viscosity and l = mixing length. For the mixing length the following expression was adopted (Pezzinga, 1999):

$$\frac{l}{R_0}=\kappa\frac{y}{R_0}e^{-(y/R_0)} \tag{4.78}$$

Where y = distance from wall, R_0 = pipe radius; and κ = constant. The following expression (Pezzinga, 1999) is used to obtain κ.

$$\kappa=0.374+0.0132\ln(1+\frac{83{,}100}{\Re}) \tag{4.79}$$

The value of κ was evaluated with initial Reynolds number and maintained constant throughout the simulation.

4.7.2 Evaluating Thickness of Viscous Sub layer

Thickness of viscous sub layer δ is distance from wall to the intersection between the velocity profiles in the viscous sub layer and the turbulent zone. Linear velocity profile is assumed in the viscous sub layer and in the turbulent zone the profile is locally logarithmic.

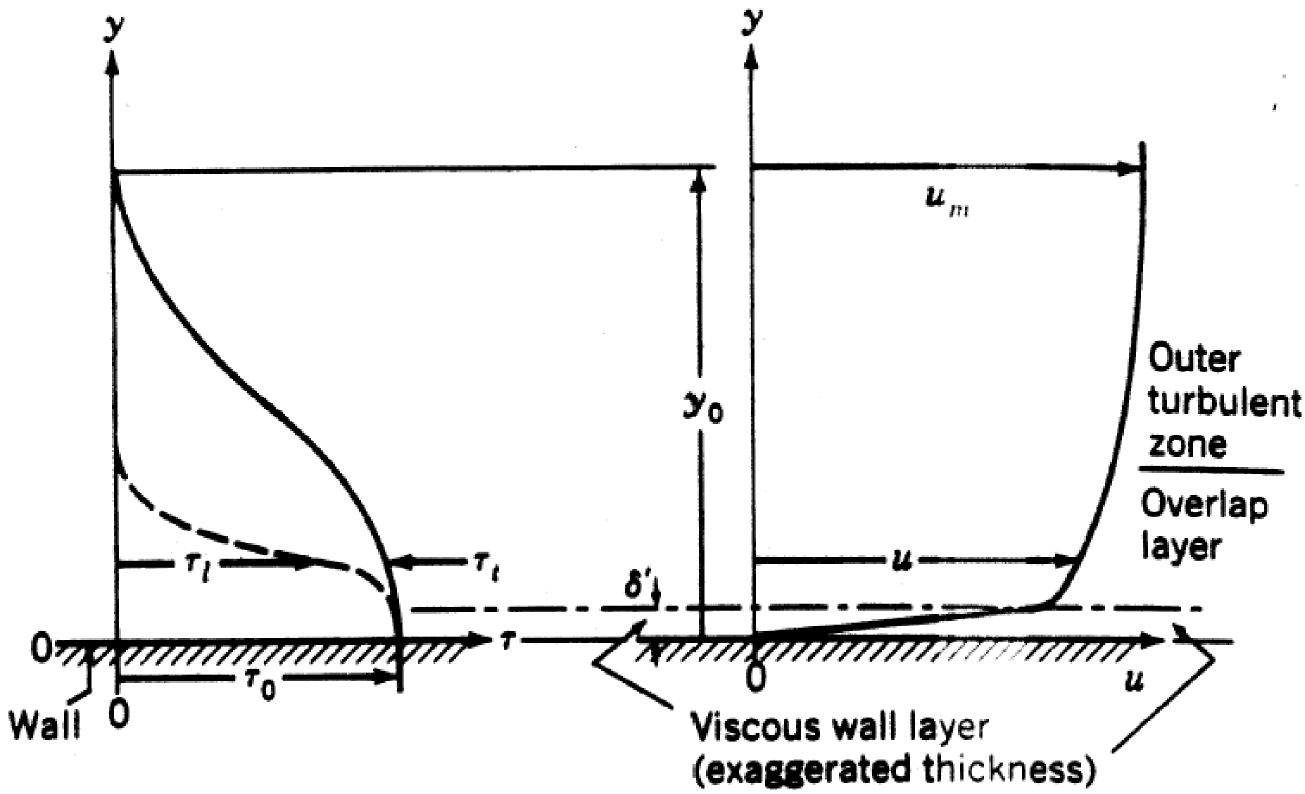


Figure 12. Shear stress distribution and velocity profile for turbulent flow (reproduced from Wylie et al., 1993)

If $u_* = \sqrt{\tau_w / \rho}$ is the friction velocity and τ_w is the wall shear stress then the equation for velocity profile in the viscous sub layer is (Pezzinga, 1999):

$$\frac{u}{u_*} = \frac{u_* y}{\nu} \tag{4.80}$$

The logarithmic velocity profile in turbulent zone is (Pezzinga G., 1999)

$$\frac{u}{u_*} = 2.5 \ln \frac{y}{\varepsilon} + 5.0 \qquad (4.81)$$

Viscous sub layer thickness is obtained from the following expression (Streeter and Wylie 1985)

$$\delta = 11.63\nu / u_* \qquad (4.82)$$

4.7.3 Boundary Conditions

With regard to the wall condition, if first grid mesh close to the wall is in viscous sub layer then Eq. 4.80 is used to calculate the wall shear stress τ_w otherwise, Eq. 4.81 is used.

On the pipe axis, symmetry condition is adopted. The initial velocity profile was determined by an iterative procedure, using the unsteady-flow momentum equation and constant discharge. The initial values of discharge in the pipe and heads at the nodes were obtained from steady-state conditions.

4.8 Finite Difference Scheme

The pipe is divided into cylindrical grid elements of constant length Δx in longitudinal direction and constant area ΔA in radial direction. Velocities are defined at center of each radial mesh, and shear stresses on internal and external sides. Other parameters such as pressure head H, mass m, Temperature T, ϕ and s vary along longitudinal direction only and are defined at each grid point.

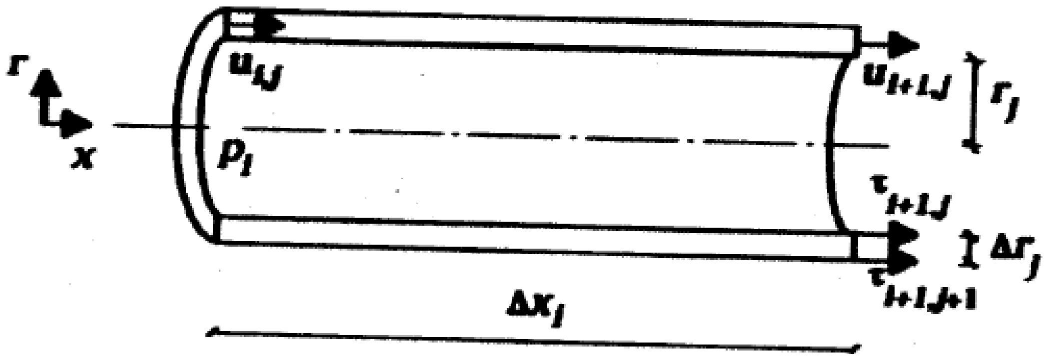


Figure 13. Cylindrical grid element (reproduced from Cannizzaro and Pezzinga, 2005)

The time step Δt is defined as: $\Delta t = C\,\Delta x/c$. Where C is the Courant number and c the wave speed of the fluid.

4.8.1 Mac Cormack Method

MacCormack finite difference method (MacCormack, 1969) is used in this model. This technique is second order accurate in both space and time. In the *predictor* step, forward finite differences are used (Cannizzaro and Pezzinga, 2005):

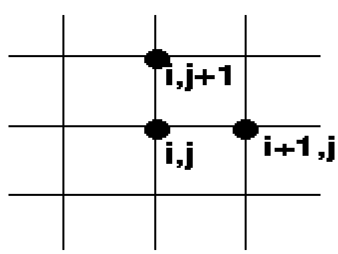


Figure 14. 2-D Finite difference grid (reproduced from Anderson, 1995)

Predictor Step:

$$\frac{m_i^p - m_i^{n-1}}{\Delta t} = \frac{V_l M}{\Delta t}\frac{1}{K_H^{'} RT}(p_{si}^{n-1} - p_i^{n-1}) \qquad (4.83)$$

$$\frac{\phi_i^p - \phi_i^{n-1}}{\Delta t} + \frac{c^2}{gA}\frac{Q_{i+1}^{n-1} - Q_i^{n-1}}{\Delta x} = 0 \qquad (4.84)$$

$$\frac{s_i^p - s_i^{n-1}}{\Delta t} - c_P K_N \left(\frac{T_0}{T_i^{n-1}} - 1\right) = 0 \tag{4.85}$$

$$\rho \frac{u_{i,j}^p - u_{i,j}^{n-1}}{\Delta t} + \frac{p_{i+1}^{n-1} - p_i^{n-1}}{\Delta x} + 2\Pi \frac{(r_{j+1}\tau_{i,j+1}^* - r_j \tau_{i,j}^*)}{\Delta A} = 0 \tag{4.86}$$

Corrector Step:

Backward finite differences are used in the *corrector* step (Cannizzaro and Pezzinga, 2005):

$$\frac{m_i^c - m_i^{n-1}}{\Delta t} = \frac{V_l M}{\Delta t} \frac{1}{K_H^{'} RT} (p_{si}^p - p_i^p) \tag{4.87}$$

$$\frac{\phi_i^c - \phi_i^{n-1}}{\Delta t} + \frac{c^2}{gA} \frac{Q_i^p - Q_{i-1}^p}{\Delta x} = 0 \tag{4.88}$$

$$\frac{s_i^c - s_i^{n-1}}{\Delta t} - c_P K_N \left(\frac{T_0}{T_i^p} - 1\right) = 0 \tag{4.89}$$

$$\rho \frac{u_{i,j}^c - u_{i,j}^{n-1}}{\Delta t} + \frac{p_i^p - p_{i-1}^p}{\Delta x} + 2\Pi \frac{(r_{j+1}\tau_{i,j+1}^* - r_j \tau_{i,j}^*)}{\Delta A} = 0 \tag{4.90}$$

In above equations, indices i, j, and n refer, respectively, to directions x, and r, and time t. Indices p and c refer to predictor and corrector steps. $\tau_{i,j}^*$ is average between values of stress in previous and in current time steps. The value at time n is computed as the average of predictor and corrector values.

The flowchart below explains the gaseous cavitation problem modeled in MATLAB:

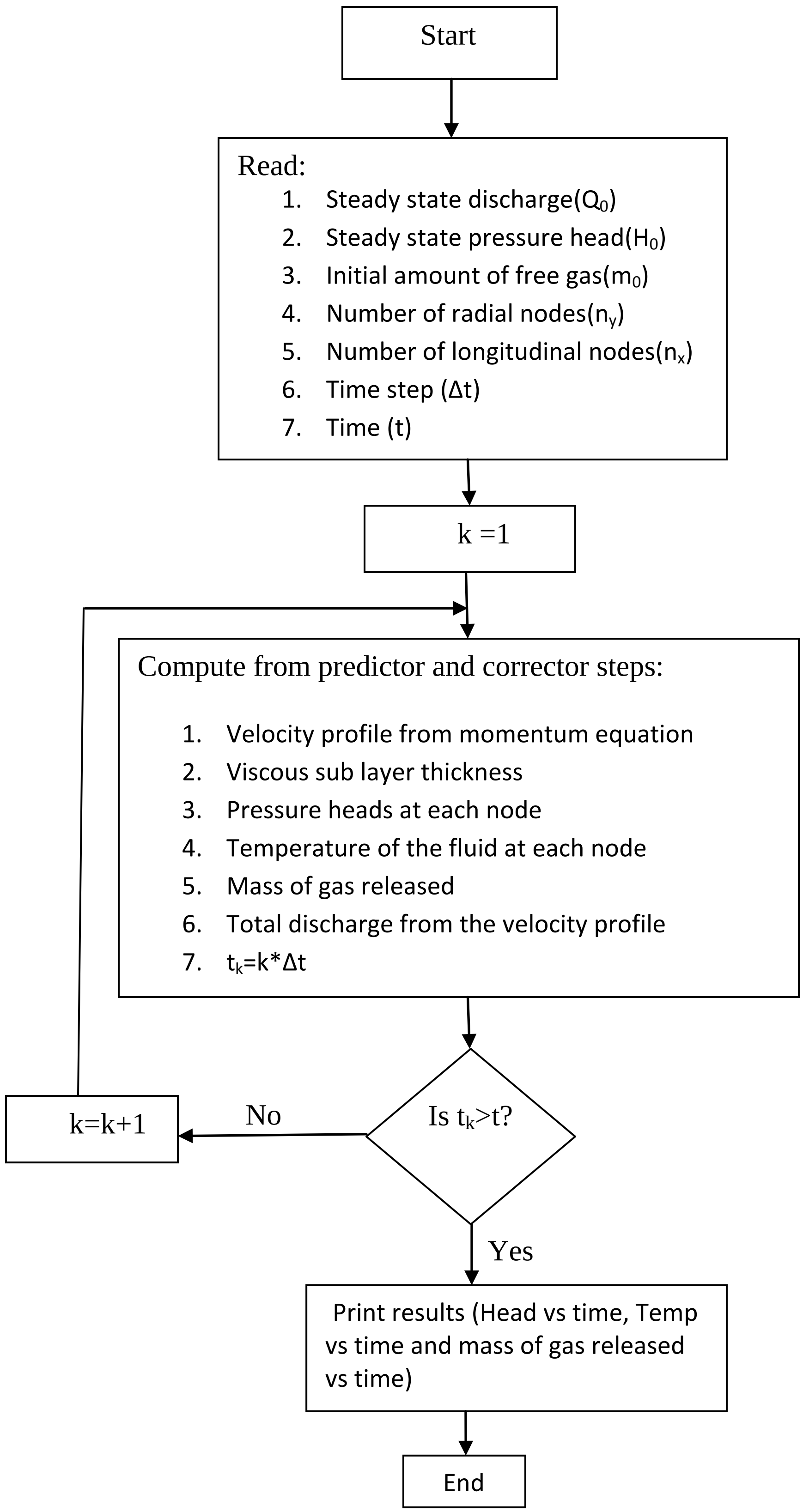


Figure 15. Flowchart to show steps for solving gaseous cavitation

4.9 Application of the model

The above model is applied to the single pipe system setup as shown in Figure 6 in Chapter 2. Reservoir is at upstream of the pipe and the valve present downstream is closed suddenly. An initial amount of free gas (air) is assumed in the pipe along with water. Below is the set of values used in the problem:

Table 15. Data used by gaseous cavitation model

Steady state Discharge	0.227*10^-3 cu.m/s
Pipe diameter	53.9*10^-3 m
Kinematic Viscosity of water	0.837 * 10^-6 m/s2
Pipe roughness	0.1*10^-3
Initial mass of free gas	4.5*10^-6 kg/cu.m
Initial temperature of water	28 deg.C
Wave speed	1324 m/s
Length of the pipe	36m
Reservoir Head	4.53 m
Time Interval	0.002 sec

A detailed descitption of all the parameters used in the model can be found in Appendix D.

The following matlab results are obtained:

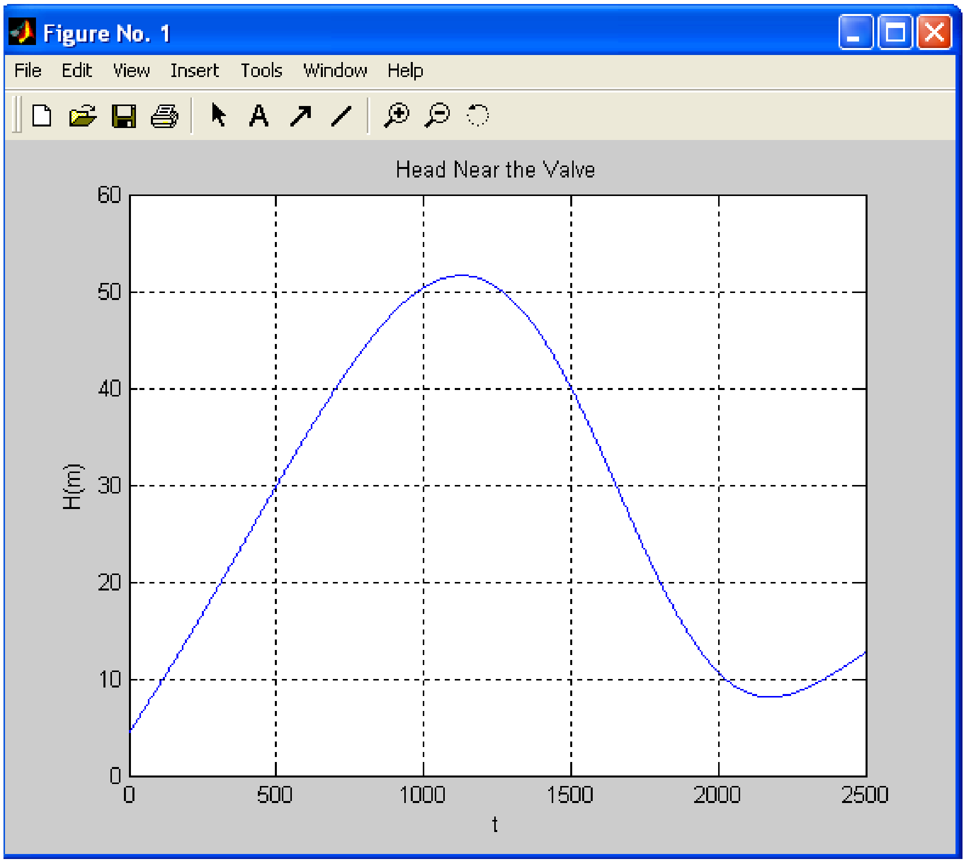


Figure 16. Head vs Time plot near the valve

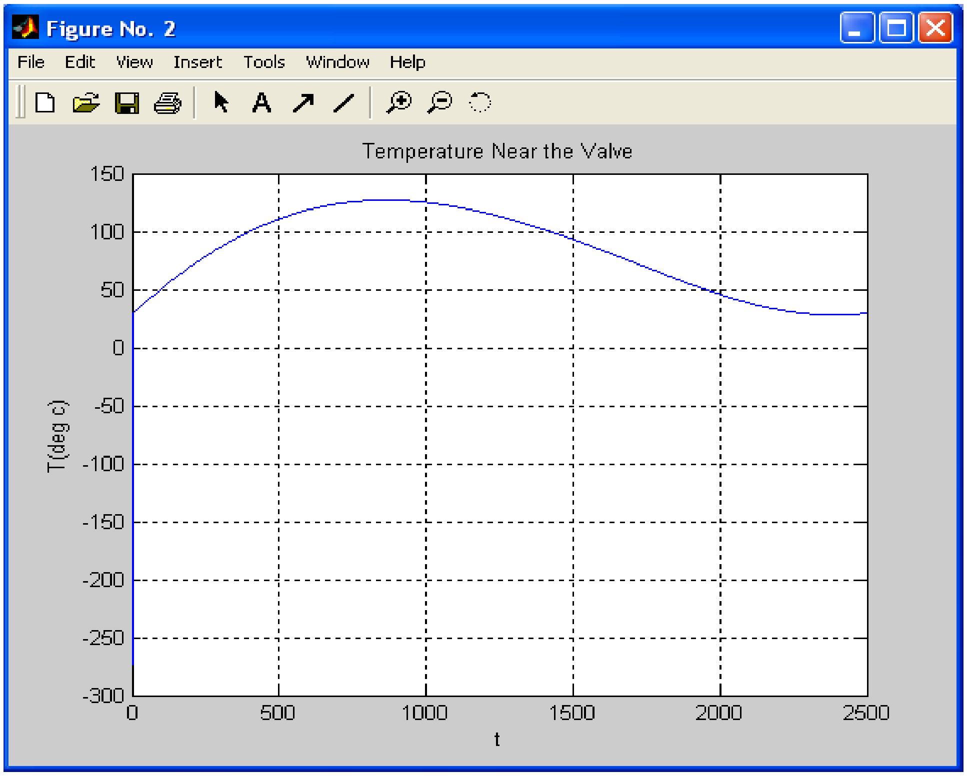


Figure 17. Temperature vs Time plot near the valve

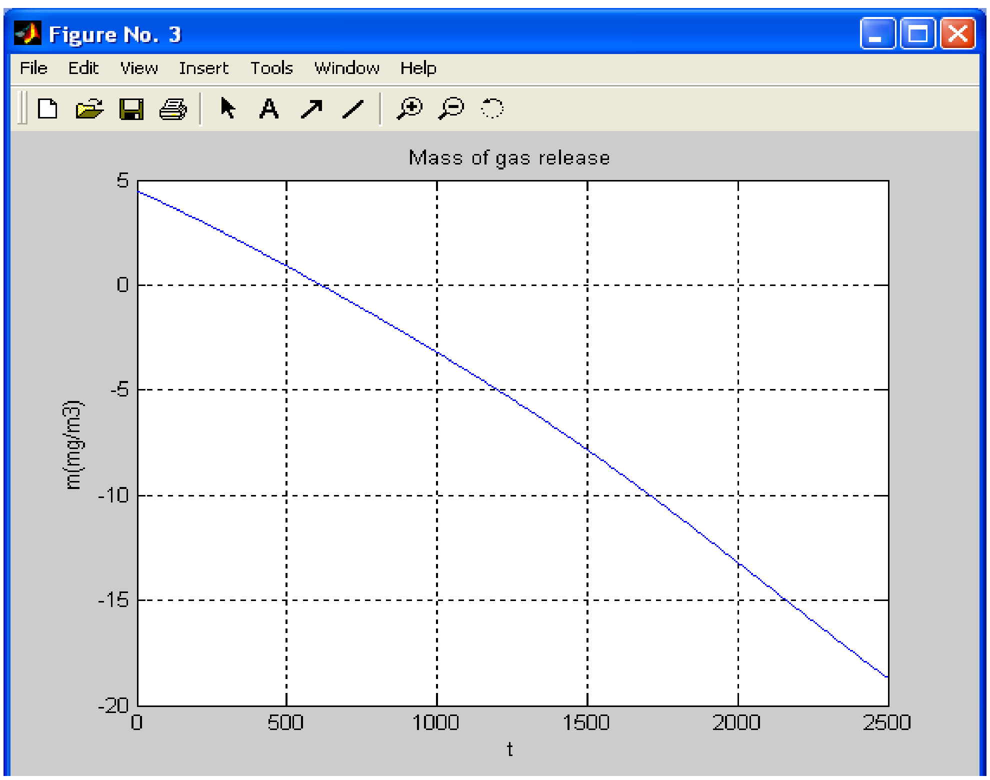


Figure 18. Mass of released gas vs time plot near the valve

As shown above, the results representing head and temperature near the valve, and total mass of gas release are plotted for 2500 timesteps (i.e., 5 s). As shown in figure 16, a maximum pressure head of 52 m is reached near the valve during the first 5 s. Also from figure17, a peak temperature of 140 (deg c) is attained during the first 5 s resulting in vaporization of liquid near the valve. As shown in figure 18, the total amount of free gas is reduced during this time. These results can be verified by conducting an experiment with similar conditions.

The peak value for head near the valve is attained only once, and further oscillation in pressure head cannot be plotted. This may be due to stability and concurrency issues which are part of numerical schemes. By using a smaller time step, one can increase the stability of the numerical computations. But, as MacCormack method is an explicit finite difference method, time steps cannot be smaller than a certain value decided by the stability criteria or else, the resulting solution will not be accurate.

It can be observed that high values of temperature can be reached during cavitation process. The combination of high values of pressure and temperature in a pipeline may give rise to disastrous consequences. Accidents arisen from operationg errors in pumping plants of combustible liquids are mentioned in literature.

Chapter 5: Summary and Conclusions

5.1 Summary

In this thesis, first one dimensional flow in closed conduits was considered to analyse the changes in pressure and velocity in water distribution system during transient conditions. Equations related to method of characteristics and finite difference approximations are discussed. Various boundary conditions such as reservoir, valve, pumps/turbines, and junctions are considered in the pipe system.

In order to demonstrate the use of method of characterisitics for transient analysis two problems are solved.

1) A single pipe with reservoir upstream and open valve downstream is considered. In order to simulate transients, the valve is closed suddenly. An excel spreadsheet is developed to calculate pressure and velocity heads at each time step. The maximum pressure heads are then compared with Joukowski's equation and the results were found comparable. The excel spreadsheet can be used by any researcher to estimate the critical pressures reached when a valve is closed suddenly and to measure the total time it takes for pressure waves to die down and for water to become steady.

2) In order to perform transient analysis in pipe network, a distribution system with six pipes, three reservoirs, a pump located upstream of one of the reservoirs and a valve at downstream of one of the pipes is considered. In this network there is a constant demand at three junctions. Valve is closed suddenly to simulate transient analysis and variations in pressure and velocity heads are studied in each of the connecting pipes. TRANSNET program is used for transient analysis and results are compared with WHAMO which gave comparable results for pressure and velocity head values in pipes.

In Chapter 3, a transient analysis program is developed in Java. This program can analyze transients caused by suddenly-closing valves, gradually-closing valves, pump power failures and sudden demand changes at junctions. A maximum of four pipes can be present at a junction. This program is used to solve the pipe network problem discussed above; the results were found to be similar to that of TRANSNET program.

In Chapter 4 a two – dimensional numerical model is developed using MATLAB to analyse gaseous cavitation in pipes. Two phase flow is assumed, with free gas being distributed throughout the pipe as a homogeneous fluid mixture with gas bubbles and liquid moving at same velocity. Equations used in the model such as Henry's law, energy equation of a gas, mixture continuity and momentum equations are derived and adopted from literature.

The results from the model show that during transients, there is significant increase in fluid temperature along with high pressures. As a result some of the fluid gets vaporized and pockets of air are formed in distribution systems. In literature pipe failures and noise problems in premise plumbing are atributed to gaseous cavitation.

5.2 Conclusion

In this study, an excel spreadsheet is developed to critically analyse transients that can occur in closed conduits. The Java program developed as part of this work is an attempt to introduce object oriented technology for analyzing problems in hydraulic engineering field. The code can be further extended, for example by developing java applets and graphical user interphase to make it more user friendly. A MATLAB program was developed to analyse gaseous cavitation using standard equations proposed by Cannizzaro and Pezzinga (2005). Though the gaseous cavitation program seemed stable, there are issues with accuracy and concurrency of the code. Based on author's experience in this work, it is recommended to use separate models for gas release and for thermic exchange so that parameters can be better estimated.

Appendix A-1: Steady state analysis results

```
STEADY STATE ANALYSIS OF THE NETWORK
 ------------------------------------
 Example Problem 2

 ALL DEMAND FLOWS ARE MULTIPLIED BY    1.0000

 PIPES           6
 NODES           6
 SOURCE PUMPS    1
 BOOSTER PUMPS   0
 RESERVOIRS      2
 MINOR LOSSES    0
 PRVS            0
 NOZZLES         0
 CHECK VALVE     0
 BACK PRES. V.   0
 DIF. HEAD DEV   0
 SPECIFIED Q     0
 SPECIFIED PRES  0
 NODES AT S. P. & RES. WHICH HAVE BEEN ELIMINATED
    4    5    6

 PIPE  2ND ORDER COEF  LINEAR COEF  SHUT-OFF HEAD  SUMP ELEV
    6           -.504        1.122          97.00    4130.00

 RES.(NOZZLE) PIPES & THEIR ELEV. ARE
    1  4200.0   -5  4130.0
 N9=    6 N8=    3

 JUNCTION EXT. FLOW PIPES AT JUNCTION
    1    1     1.058       -1    -2     3
    2    2      .706        2     4    -6
    3    3     1.760       -3    -4    -5

 FLOW FROM PUMPS AND RESERVOIRS EQUALS  1582.000
```

```
ITERATION=    1 SUM=  .740E+01
ITERATION=    2 SUM=  .315E+01
ITERATION=    3 SUM=  .903E+00
ITERATION=    4 SUM=  .108E+00
ITERATION=    5 SUM=  .587E-02
ITERATION=    6 SUM=  .190E-04

THE FLOWRATE     3.787 IS QUITE DIFFERENT THAN THE NORMAL CAPACITY     6.684
AS GIVEN BY THE PUMP CHARACTERISTICS FOR PUMP   1 IN PIPE    6
HAVE YOU ERRED.

PUMPS:
 NODE PIPE    HEAD     FLOW      HORSEPOWER      KILOWATTS   KWATT-HRS/DAY
    6    6    94.03   1699.93       40.40          30.14       723.38
```

RESULTS OF SOLUTION

PIPE NO.	FLOWRATE (CFS)	(GPM)	HEAD LOSS	K IN EXP. FORMULA	DIA. FEET	LENGTH FEET	JUN. NO	HEAD FEET	PRESSURE (PSF)	JUN. NO	HEAD FEET	PRESSURE (PSF)	PRESSURES (PSI)	
1	.758	340.2	1.317	.220E+01	1.00	3300.0	0	4200.00	0.	1	398.68	24878.	.000	172.762
2	.608	273.0	15.698	.394E+02	.67	8200.0	2	384.38	239.85	1	398.68	24878.	166.565	172.762
3	.308	138.1	1.789	.159E+02	.67	3300.0	1	398.68	248.78	3	826.89	51598.	172.762	358.321
4	2.473	1110.0	17.487	.327E+01	1.00	4900.0	2	384.38	239.85	3	826.89	51598.	166.565	358.321
5	-1.021	-458.1	66.894	.644E+02	.50	3300.0	0	4130.00	0.	3	826.89	51598.	.000	358.321
6	3.787	1700.0	9.645	.819E+00	1.17	2600.0	0	4130.00	0.	2	384.38	23985.	.000	166.565

UNITS OF SOLUTION ARE
DIAMETERS - inch
LENGTH - feet
HEADS - feet
ELEVATIONS - feet
PRESSURES - (psi)
FLOWRATES - (gpm)
HAZEN-WILLIAMS FORMULA USED FOR COMPUTING HEAD LOSS

PIPE DATA

PIPE NO.	NODES FROM	TO	LENGTH	DIAM	COEF	FLOW RATE	VELOCITY	HEAD LOSS	HLOSS /1000
1	4	1	3300.	12.0	120.0	340.13	.96	1.32	.40
2	2	1	8200.	8.0	120.0	272.99	1.74	15.70	1.91
3	1	3	3300.	8.0	120.0	138.11	.88	1.79	.54
4	2	3	4900.	12.0	120.0	1109.95	3.15	17.49	3.57
* 5	3	5	3300.	6.0	120.0	458.06	5.20	66.89	20.27
6	6	2	2600.	14.0	120.0	1699.93	3.54	9.64	3.71

NODE DATA:

NODE NO.	DEMAND (gpm)	(cfs)	ELEV	HEAD	PRESSURE	HGL ELEV
1	475.0	1.06	3800.	398.68	172.76	4198.68
2	317.0	.71	3830.	384.38	166.57	4214.38
3	790.0	1.76	3370.	826.89	358.32	4196.89
6	-1699.9	-3.79	4010.	214.03	92.74	4224.03
4	-340.1	-.76	4050.	150.00	65.00	4200.00
5	458.1	1.02	4000.	130.00	56.33	4130.00

Appendix A-2: Transient Analysis results

```
******************************

 * NETWORK TRANSIENT ANALYSIS *
 ******************************


        DEMONSTRATION OF PROGRAM TRANSNET - INPUT DATA FILE "EPB12_5.DAT"
        NETWORK OF EXAMPLE 2 - SUDDENLY-CLOSED VALVE AT THE DS END OF PIPE 5


             IOUT = 100
           NPARTS =   4
           NPIPES =   6

             HATM = 30.0 FT

             TMAX =  20.00 SEC
             DELT =   .227 SEC



                  TRANSIENT CONDITIONS IMPOSED
                  ----------------------------
 SUDDENLY CLOSED VALVE AT DOWNSTREAM END OF PIPE    5


                    PIPE INPUT DATA
                    ---------------

   PIPE DIAM-IN LENGTH-FT WAVE SPD-FPS PIPEZ-FT C-VALUE VEL-FPS
   ---- ------- --------- ------------ -------- ------- -------
     1   12.00    3300.0       2850.     4050.    120.     .97
     2    8.00    8200.0       2850.     3830.    120.    1.74
     3    8.00    3300.0       2850.     3800.    120.     .88
     4   12.00    4900.0       2850.     3830.    120.    3.15
     5    6.00    3300.0       2850.     3370.    120.    5.20
     6   14.00    2600.0       2850.     4010.    120.    3.54
```

PIPE	DELT-SEC	PARTS	SINE	L/A-SEC	INTERPOLATION
1	.289	5	-.07576	1.16	.019
2	.718	12	-.00366	2.88	.052
3	.289	5	-.13030	1.16	.019
4	.428	7	-.09388	1.72	.075
5	.288	5	.19091	1.16	.019
6	.227	4	-.06923	.91	.004

NODE INPUT DATA

NODE	ELHGL-FT	GRNDEL-FT	PIPES 1	PIPES 2	PIPES 3	PIPES 4	DEMAND-CFS
1	4198.68	3800.00	-1	-2	3	0	1.06
2	4214.38	3830.00	2	4	-6	0	.71
3	4196.89	3370.00	-3	-4	5	0	1.76
6	4224.03	4010.00	6	0	0	0	-3.79
4	4200.00	4050.00	1	0	0	0	-.76
5	4130.00	4000.00	-5	0	0	0	1.02

** PUMP INFORMATION **

		Q-GPM	HEAD/STAGE-FT	HP/STAGE-HP
LINE	= 6	.00	118.0	57.0
PUMPS	= 1	2000.76	92.0	68.0
STAGES	= 1	3001.14	82.0	77.0
RPM	= 1180. RPM	4001.52	67.0	80.0
SUMP ELEV	= 4130. FT	4501.70	52.0	76.0
WRSQ	= 50. LB-FTSQ	5302.01	.0	60.0

PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME

TIME = .000 SEC

		X/L	HEAD-FT	H-FT	V-FPS	X/L	HEAD-FT	H-FT	V-FPS
PIPE	1	.000	150.	4200.	.97	.200	200.	4200.	.97
		.400	249.	4199.	.97	.600	299.	4199.	.97
		.800	349.	4199.	.97	1.000	399.	4199.	.97
PIPE	2	.000	384.	4214.	1.74	.083	386.	4213.	1.74
		.167	387.	4212.	1.74	.250	388.	4210.	1.74
		.333	389.	4209.	1.74	.417	390.	4208.	1.74
		.500	391.	4206.	1.74	.583	393.	4205.	1.74
		.667	394.	4204.	1.74	.750	395.	4202.	1.74
		.833	396.	4201.	1.74	.917	397.	4200.	1.74
		1.000	398.	4198.	1.74				
PIPE	3	.000	399.	4199.	.88	.200	484.	4198.	.88
		.400	570.	4198.	.88	.600	656.	4198.	.88
		.800	741.	4197.	.88	1.000	827.	4197.	.88
PIPE	4	.000	384.	4214.	3.15	.143	448.	4212.	3.15
		.286	511.	4209.	3.15	.429	574.	4207.	3.15
		.571	637.	4204.	3.15	.714	700.	4202.	3.15
		.857	764.	4199.	3.15	1.000	827.	4197.	3.15
PIPE	5	.000	827.	4197.	5.20	.200	687.	4183.	5.20
		.400	548.	4170.	5.20	.600	408.	4156.	5.20
		.800	269.	4143.	5.20	1.000	129.	4129.	5.20
PIPE	6	.000	214.	4224.	3.54	.250	257.	4222.	3.54
		.500	299.	4219.	3.54	.750	342.	4217.	3.54
		1.000	384.	4214.	3.54				

PIPE 6 PUMP SPEED = 1180.0 RPM PUMP DISCHARGE = 1700.6 GPM EACH
PUMP HEAD = 94.0 FT

COLUMN SEPARATION HAS OCCURRED AT 7.73 SEC IN PIPE 5 AT LOCATION 1.000

TIME = 7.726 SEC

	X/L	HEAD-FT	H-FT	V-FPS	X/L	HEAD-FT	H-FT	V-FPS
PIPE 1	.000	150.	4200.	-.25	.200	201.	4201.	-.25
	.400	268.	4218.	-.40	.600	332.	4232.	-.47
	.800	375.	4225.	-.25	1.000	407.	4207.	.04
PIPE 2	.000	379.	4209.	1.63	.083	335.	4163.	1.48
	.167	299.	4124.	1.17	.250	272.	4094.	1.11
	.333	240.	4060.	1.12	.417	228.	4046.	1.34
	.500	223.	4038.	1.49	.583	223.	4035.	1.59
	.667	228.	4038.	1.69	.750	267.	4074.	1.58
	.833	316.	4121.	1.71	.917	367.	4170.	2.15
	1.000	407.	4207.	2.61				
PIPE 3	.000	407.	4207.	-.33	.200	476.	4190.	-.22
	.400	556.	4184.	-.22	.600	646.	4188.	-.22
	.800	735.	4191.	-.21	1.000	822.	4192.	-.23
PIPE 4	.000	379.	4209.	2.60	.143	454.	4219.	2.66
	.286	516.	4215.	2.69	.429	564.	4197.	2.45
	.571	619.	4186.	2.19	.714	690.	4191.	2.13
	.857	757.	4193.	2.10	1.000	822.	4192.	2.07
PIPE 5	.000	822.	4192.	-1.10	.200	693.	4189.	-1.14
	.400	493.	4115.	-.56	.600	306.	4054.	-.35
	.800	120.	3994.	-.27	1.000	-40.	3960.	.00
PIPE 6	.000	219.	4229.	3.07	.250	260.	4225.	3.07
	.500	301.	4221.	3.07	.750	340.	4215.	3.09

```
                 1.000   379.  4209.    3.10

PIPE   6     PUMP SPEED = 1180.0 RPM     PUMP DISCHARGE = 1471.4 GPM EACH
                                         PUMP HEAD =  98.9 FT
```

```
                         ***************************
                         * TABLE OF EXTREME VALUES *
                         ***************************
```

	X	MAX HEAD	TIME	MIN HEAD	TIME	MAX H	MIN H
PIPE 1							
	.000	150.0	7.7	150.0	7.7	4200.	4200.
	.200	256.1	3.6	176.7	5.9	4256.	4177.
	.400	308.8	3.9	224.3	6.1	4259.	4174.
	.600	359.7	4.1	275.5	5.9	4260.	4175.
	.800	411.9	4.3	327.6	5.7	4262.	4178.
	1.000	463.0	4.5	378.5	5.5	4263.	4179.
PIPE 2							
	.000	476.5	4.5	337.5	6.8	4306.	4167.
	.083	476.8	4.8	317.8	7.5	4304.	4145.
	.167	522.0	5.0	283.7	7.3	4347.	4109.
	.250	532.8	5.2	267.1	7.5	4355.	4090.
	.333	535.2	5.5	240.2	7.7	4355.	4060.
	.417	537.1	5.7	228.3	7.7	4355.	4046.
	.500	539.0	5.9	223.3	7.7	4354.	4038.
	.583	538.2	5.7	222.8	7.7	4351.	4035.
	.667	535.3	5.5	227.7	7.7	4345.	4038.
	.750	512.4	5.2	266.7	7.7	4320.	4074.
	.833	459.9	5.0	316.4	7.7	4265.	4121.
	.917	461.5	4.8	367.1	7.7	4264.	4170.
	1.000	463.0	4.5	378.5	5.5	4263.	4179.
PIPE 3							
	.000	463.0	4.5	378.5	5.5	4263.	4179.
	.200	607.3	2.5	440.4	5.7	4321.	4154.
	.400	697.2	2.7	514.0	5.9	4325.	4142.
	.600	785.3	3.0	597.7	6.1	4327.	4140.
	.800	874.9	3.2	687.6	5.9	4331.	4144.
	1.000	963.1	3.4	781.0	6.6	4333.	4151.
PIPE 4							
	.000	476.5	4.5	337.5	6.8	4306.	4167.
	.143	555.4	3.0	392.0	5.5	4320.	4156.
	.286	635.8	3.2	443.8	5.5	4334.	4142.
	.429	702.4	3.4	504.5	5.7	4335.	4137.

	.571	768.8	3.6	567.2	5.9	4336.	4134.
	.714	835.0	3.9	631.5	6.1	4336.	4133.
	.857	899.1	3.6	717.4	6.6	4335.	4153.
	1.000	963.1	3.4	781.0	6.6	4333.	4151.
PIPE 5							
	.000	963.1	3.4	781.0	6.6	4333.	4151.
	.200	1119.2	1.4	578.5	3.6	4615.	4074.
	.400	1001.5	1.6	439.7	3.9	4623.	4062.
	.600	882.0	1.8	305.9	7.7	4630.	4054.
	.800	762.4	2.0	120.0	7.7	4636.	3994.
	1.000	642.8	2.3	-39.7	7.7	4643.	3960.
PIPE 6							
	.000	227.4	5.9	214.0	.0	4237.	4224.
	.250	329.6	3.9	224.9	6.1	4295.	4190.
	.500	386.3	4.1	256.0	6.4	4306.	4176.
	.750	431.6	4.3	296.1	6.6	4307.	4171.
	1.000	476.5	4.5	337.5	6.8	4306.	4167.

MAXIMUM HEAD =1119.2 FT IN PIPE 5 AT X = .200 AT TIME = 1.36 SEC

MINIMUM HEAD = -39.7 FT IN PIPE 5 AT X =1.000 AT TIME = 7.73 SEC

```
**************************************************
* HEAD AND VELOCITY VS TIME FOR SELECTED POINTS *
**************************************************
```

PIPE 5 NODE 5

TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS
.00	129.3	5.20	.23	589.8	.00	.45	590.5	.00
.68	603.3	.00	.91	604.2	.00	1.14	616.7	.00
1.36	617.3	.00	1.59	629.1	.00	1.82	630.0	.00
2.05	641.5	.00	2.27	642.8	.00	2.50	175.5	.00
2.73	89.5	.00	2.95	73.1	.00	3.18	71.6	.00
3.41	64.0	.00	3.64	63.5	.00	3.86	58.2	.00
4.09	58.9	.00	4.32	52.3	.00	4.54	51.8	.00
4.77	258.4	.00	5.00	336.3	.00	5.23	355.9	.00
5.45	359.0	.00	5.68	342.8	.00	5.91	317.5	.00
6.14	303.1	.00	6.36	294.8	.00	6.59	295.0	.00
6.82	294.4	.00	7.04	158.8	.00	7.27	82.3	.00
7.50	17.1	.00	7.73	-39.7	.00	7.95	.0	.00

PIPE 5 NODE 3

TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS
.00	426.9	5.20	.23	426.8	5.20	.45	426.8	5.19
.68	426.7	5.19	.91	426.7	5.19	1.14	426.7	5.19
1.36	540.8	-2.26	1.59	551.9	-2.99	1.82	554.4	-3.09
2.05	554.8	-3.10	2.27	556.6	-3.16	2.50	557.0	-3.16
2.73	559.8	-3.20	2.95	560.8	-3.19	3.18	562.6	-3.24
3.41	563.1	-3.24	3.64	478.6	.38	3.86	454.8	1.42
4.09	450.0	1.63	4.32	449.5	1.65	4.54	436.4	1.57
4.77	422.7	1.42	5.00	413.4	1.36	5.23	409.7	1.31
5.45	407.3	1.34	5.68	406.6	1.34	5.91	421.7	-.55
6.14	428.9	-1.46	6.36	408.0	-1.97	6.59	381.0	-2.32

6.82	382.4	-2.13	7.04	398.9	-1.66	7.27	412.7	-1.33
7.50	419.3	-1.15	7.73	422.3	-1.10	7.95	.0	.00

PIPE 1 NODE 1

TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS
.00	398.7	.97	.23	398.6	.97	.45	398.6	.97
.68	398.6	.97	.91	398.6	.97	1.14	398.6	.97
1.36	398.5	.97	1.59	398.5	.97	1.82	398.5	.97
2.05	398.5	.97	2.27	398.6	.97	2.50	447.0	.42
2.73	456.3	.32	2.95	458.1	.30	3.18	458.3	.30
3.41	459.2	.29	3.64	459.4	.29	3.86	461.0	.27
4.09	461.8	.26	4.32	462.7	.26	4.54	463.0	.25
4.77	406.2	.00	5.00	384.7	-.09	5.23	379.4	-.11
5.45	378.5	-.12	5.68	382.6	-.18	5.91	388.6	-.25
6.14	391.1	-.31	6.36	392.1	-.34	6.59	391.5	-.35
6.82	391.2	-.35	7.04	421.9	-.38	7.27	439.4	-.40
7.50	428.0	-.21	7.73	407.1	.04	7.95	.0	.00

PIPE 6 NODE 2

TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS	TIME-SEC	HEAD-FT	V-FPS
.00	384.3	3.54	.23	384.3	3.54	.45	384.2	3.54
.68	384.2	3.54	.91	384.1	3.54	1.14	385.8	3.56
1.36	385.8	3.56	1.59	385.8	3.57	1.82	385.8	3.57
2.05	385.8	3.56	2.27	385.9	3.56	2.50	385.9	3.56
2.73	385.9	3.56	2.95	431.0	3.06	3.18	459.0	2.74
3.41	469.6	2.63	3.64	472.5	2.60	3.86	474.1	2.59
4.09	474.7	2.58	4.32	475.8	2.58	4.54	476.5	2.58
4.77	442.4	2.15	5.00	420.0	1.87	5.23	384.9	2.07
5.45	361.2	2.29	5.68	350.6	2.38	5.91	347.0	2.41
6.14	344.4	2.43	6.36	342.3	2.45	6.59	339.3	2.45
6.82	337.5	2.45	7.04	357.3	2.70	7.27	374.0	2.91
7.50	378.9	3.04	7.73	378.8	3.10	7.95	.0	.00

Appendix B: Comparison between WHAMO and TRANSNET results

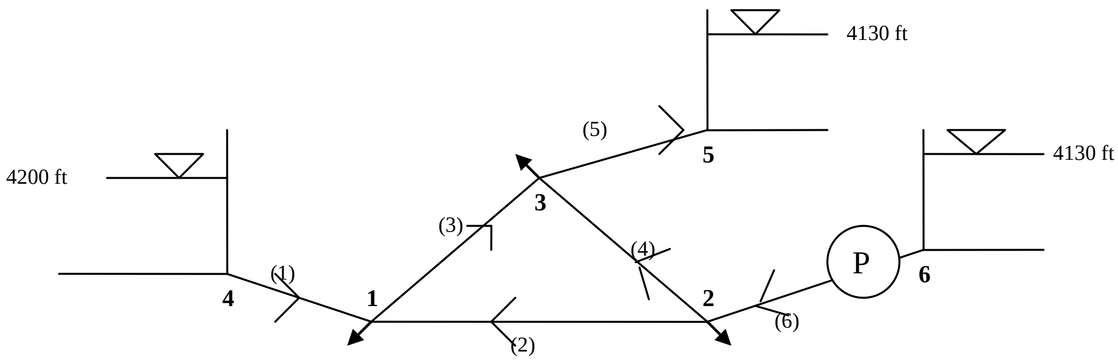


Figure A.1 Network distribution system

(1) Comparing the results at steady state condition:

Time = 0.0 sec
Total head (ft)

Node	WHAMO	Transient Program
1	4200	4199
2	4223	4214
3	4202	4197
4	4200	4200
5	4130	4129
6	4234	4224

Discharge(cfs)

Pipe	WHAMO	Transient Program
1	0	0.8
2	0.7	0.6
3	-0.3	0.3
4	2.7	2.5
5	0.6	1.0
6	4.2	3.8

(2) At time t = 7.73 sec, (just at the time of column separation noted by transient program)

Time =7.73 sec

Total head(ft)

Node		WHAMO	Transient Program
1		4207	4207
2		4227	4209
3		4223	4192
4		4200	4200
5		4130	**3961**
6		4244	4229

Discharge(cfs)

Pipe	Node	WHAMO	Transient Program
1	4	-0.9	-0.2
2	2	0.7	0.6
3	1	-0.7	-0.1
4	2	2.6	2.0
5	3	0.1	-0.2
6	6	3.9	3.3

(3) Comparing maximum and minimum heads (ft):

Max heads:

Node	WHAMO	Time	Transient Program	Time
1	4258.1	4	4263	4.5
2	4291.8	4.2	4307	4.5
3	4319.1	2.6	4333	3.4
4	4200	0	4200	0
5	4656.6	1.6	4643	2.3
6	4274.4	5.1	4237	5.9

Min heads:

Node	WHAMO	Time	Transient Program	Time
1	4141	9.3	4179	5.5
2	4199	0.1	4167	6.8
3	4155	0.1	4151	6.6
4	4200	0	4200	0
5	4022	4.0	3961	7.7
6	4220	0.1	4224	0.0

Appendix C-1: Java Program input

```
4 100 30 120 0 false true true false false false true
0
0
1 -5
0
6 1 6
1 4198.68 3800
2 4214.38 3830
3 4196.89 3370
4 4200.00 4050
5 4130.00 4000
6 4224.03 4010
1 12 3300 120 0.96 4 1 2850
2 8 8200 120 1.74 2 1 2850
3 8 3300 120 0.88 1 3 2850
4 12 4900 120 3.15 2 3 2850
5 6 3300 120 5.20 3 5 2850
6 14 2600 120 3.54 6 2 2850
-1 -2 3 0
2 4 -6 0
-3 -4 5 0
1 0 0 0
-5 0 0 0
6 0 0 0
1 6 1 1 1180 4130 50
0.00 118.0 57.0 2000.76 92.0 68.0 3001.14 82.0 77.0 4001.52 67.0 80.0
4501.70 52.0 76.0 5302.01 0.0 60.0
3 1
3 1 2 5
3 4 1 5
```

Appendix C-2: Java Program Output

```
NETWORK TRANSIENT ANALYSIS
TITLE 1
TITLE 2
IOUT=100
NPARTS=4
NPIPES=6
HATM=30.0
TMAX=120.0
DELT=0.227


TRANSIENT CONDITIONS IMPOSED
------------------------------
SUDDENLY CLOSED VALVE AT DOWNSTREAM END OF PIPE 5

PIPE INPUT DATA
---------------
PIPE DIAM-IN LENGTH-FT WAVE SPD-FPS PIPEZ-FT F-VALUE VEL-FPS
---- ------- --------- ----------- -------- ------- -------
1 12.0 3300.0 2850.0 4050.0 0.028 0.96
2 8.0 8200.0 2850.0 3830.0 0.027 1.74
3 8.0 3300.0 2850.0 3800.0 0.03 0.88
4 12.0 4900.0 2850.0 3830.0 0.023 3.15
5 6.0 3300.0 2850.0 3370.0 0.024 5.2
6 14.0 2600.0 2850.0 4010.0 0.022 3.54

PIPE DELT-SEC PARTS SINE L/A-SEC INTERPOLATION
---- -------- ----- ---- ------- -------------
1 0.289 5 -0.076 1.158 0.019
2 0.716 12 -0.0040 2.877 0.052
3 0.289 5 -0.13 1.158 0.019
4 0.428 7 -0.094 1.719 0.075
5 0.282 5 0.191 1.158 0.019
6 0.227 4 -0.069 0.912 0.0030


NODE INPUT DATA
---------------

NODE  ELHGL-FT  GRNDEL-FT  1  2  3  4    DEMAND-CFS
----  --------  ---------  -- -- -- --   ----------
1 4198.68 3800.0 -1 -2 3 0  1.054
2 4214.38 3830.0 2 4 -6 0  0.703
3 4196.89 3370.0 -3 -4 5 0  1.76
4 4200.0 4050.0 1 0 0 0  -0.754
5 4130.0 4000.0 -5 0 0 0  1.021
6 4224.03 4010.0 6 0 0 0  -3.784


PUMP INFORMATION
----------------

                    Q-GPM      HEAD/STAGE-FT     HP/STAGE-HP
                    -----      -------------     -----------
LINE = 6               0.0  8.474576271186441E-5  1.8220609699376222E-4
PUMPS = 1              0.0037763013853761656  6.607296753806377E-5
2.1736867711536547E-4
```

```
STAGES = 1             0.005664452078064248  5.889112324044815E-5
2.461380608512227E-4
RPM = 1180.0 RPM       0.007552602770752331  4.811835679402471E-5
2.5572785542984173E-4
SUMP ELEV = 4130.0 FT     0.008496659242761693  0.0  2.4294146265834964E-4
WRSQ = 50.0 LB-FTSQ    0.010007191121512967  0.0  1.9179589157238127E-4



PRESSURE HEADS, H-VALUES AND VELOCITIES AS FUNCTIONS OF TIME
-------------------------------------------------------------

                   X/L  HEAD-FT  H-FT  V-FPS
TIME=0.0 SEC   ---  -------  ----  -----
PIPE1
            0.0 150.0 4200.0 0.96
            0.2 199.736 4199.736 0.96
            0.4 249.472 4199.472 0.96
            0.6 299.208 4199.208 0.96
            0.8 348.944 4198.944 0.96
            1.0 398.681 4198.681 0.96


PIPE2
            0.0 384.38 4214.38 1.74
            0.083 385.561 4213.061 1.74
            0.167 386.741 4211.741 1.74
            0.25 387.922 4210.422 1.74
            0.333 389.103 4209.103 1.74
            0.417 390.284 4207.784 1.74
            0.5 391.464 4206.464 1.74
            0.583 392.645 4205.145 1.74
            0.667 393.826 4203.826 1.74
            0.75 395.006 4202.506 1.74
            0.833 396.187 4201.187 1.74
            0.917 397.368 4199.868 1.74
            1.0 398.549 4198.549 1.74


PIPE3
            0.0 398.68 4198.68 0.88
            0.2 484.319 4198.319 0.88
            0.4 569.958 4197.958 0.88
            0.6 655.597 4197.597 0.88
            0.8 741.236 4197.236 0.88
            1.0 826.875 4196.875 0.88


PIPE4
            0.0 384.38 4214.38 3.15
            0.143 447.573 4211.859 3.15
            0.286 510.766 4209.337 3.15
            0.429 573.959 4206.816 3.15
            0.571 637.152 4204.295 3.15
            0.714 700.345 4201.773 3.15
            0.857 763.538 4199.252 3.15
            1.0 826.73 4196.73 3.15


PIPE5
            0.0 826.89 4196.89 5.2
```

```
0.2 687.369 4183.369 5.2
0.4 547.847 4169.847 5.2
0.6 408.326 4156.326 5.2
0.8 268.805 4142.805 5.2
1.0 129.283 4129.283 5.2


PIPE6
0.0 214.03 4224.03 3.54
0.25 256.604 4221.604 3.54
0.5 299.178 4219.178 3.54
0.75 341.752 4216.752 3.54
1.0 384.326 4214.326 3.54


PIPE6PUMP SPEED =1180.0 RPM   PUMP DISCHARGE=1699.155 GPM EACH   PUMP
HEAD=94.03FT
HELLO 0COLUMN SEPARATION HAS OCCURRED AT 7.727 SEC IN PIPE  5 AT LOCATION1.0
EXTREME VALUES

TABLE OF EXTREME VALUES  X    MAX HEAD    TIME   MIN HEAD   TIME    MAX H
MIN H
----------------------------------------------------------------------------
------
PIPE1

0.0  150.0  7.727  150.0  7.727  4200.0  4200.0
0.2  256.285  3.636  176.509  5.909  4256.285  4176.509
0.4  308.962  3.864  224.294  6.137  4258.962  4174.294
0.6  359.754  4.091  275.405  5.909  4259.754  4175.405
0.8  411.846  4.318  327.439  5.682  4261.846  4177.439


PIPE2

0.0  476.223  4.546  337.37  6.818  4306.223  4167.37
0.083  476.507  4.773  317.544  7.5  4304.007  4145.044
0.167  521.824  5.0  283.329  7.273  4346.824  4108.329
0.25  532.657  5.227  266.928  7.5  4355.157  4089.428
0.333  535.051  5.455  239.802  7.727  4355.051  4059.802
0.417  537.049  5.682  216.269  7.727  4354.549  4033.769
0.5  538.974  5.909  214.882  7.727  4353.974  4029.882
0.583  538.19  5.682  217.084  7.727  4350.69  4029.584
0.667  535.293  5.455  227.574  7.727  4345.293  4037.574
0.75  512.421  5.227  266.698  7.727  4319.921  4074.198
0.833  459.731  5.0  293.094  7.727  4264.731  4098.094
0.917  461.264  4.773  324.458  7.727  4263.764  4126.958


PIPE3

0.0  462.825  4.546  378.335  5.455  4262.825  4178.335
0.2  607.189  2.5  439.735  5.682  4321.189  4153.735
0.4  696.87  2.727  513.464  5.909  4324.87  4141.464
0.6  784.781  2.955  597.316  6.137  4326.781  4139.316
0.8  874.243  3.182  687.47  5.909  4330.243  4143.47


PIPE4

0.0  476.223  4.546  337.37  6.818  4306.223  4167.37
```

0.143	555.559	2.955	392.31	5.455	4319.845	4156.595
0.286	635.662	3.182	444.23	5.455	4334.234	4142.801
0.429	702.231	3.409	505.066	5.682	4335.088	4137.923
0.571	768.543	3.636	567.806	5.909	4335.686	4134.949
0.714	834.646	3.864	632.219	6.137	4336.075	4133.648
0.857	898.667	3.636	717.746	6.591	4334.381	4153.46

PIPE5

0.0	962.444	3.409	780.201	6.591	4332.444	4150.201
0.2	1118.828	1.364	577.869	3.636	4614.828	4073.869
0.4	1000.988	1.591	440.55	3.864	4622.988	4062.55
0.6	881.546	1.818	295.009	7.727	4629.546	4043.009
0.8	761.691	2.046	119.601	7.727	4635.691	3993.601

PIPE6

0.0	227.294	5.909	214.03	0.0	4237.294	4224.03
0.25	330.086	3.864	225.045	6.137	4295.086	4190.045
0.5	386.569	4.091	256.424	6.364	4306.569	4176.424
0.75	431.84	4.318	296.573	6.591	4306.84	4171.573

MAXIMUM HEAD=1118.8284208516707 FT IN PIPE 1   AT X=0.2 AT TIME =1.3636676485626478 SEC

MINIMUM HEAD=-39.03264828702777 FT IN PIPE1   AT X=1.0 AT TIME=7.727450008521666 SEC

Appendix D: Matlab program

```
%% Initial parameters
% All units are in SI system

Q0 = 0.227*10^-3; % Discharge in cu.m/s
Dia = 53.9*10^-3; % Diameter of the pipe in m
Ki_Visc0= 0.837*10^-6; % Kinematic Viscosity in m/s^2
e = 0.1*10^-3; % Pipe roughness in m
DeltaT=0.002;  % Time Interval in sec
m0=4.5*10^-6; % Initial mass of gas in Kg/m^3
Theta_T=0.06; % Calibrated value of relaxation time for Temperature variation
Theta_m=400;  % Calibrated value of relaxation time for Variable Mass
Gas_Const=287.05;%Gas Constant of Air in J/Kg.K
T0_c=28;   % Temperature in Deg.C
T0_k=301.15; % Absolute Temperature in Deg.K
p_atm=1.013*10^5; % Atmospheric pressure in Pa
c=1324; % Wave speed in m/s
g=9.81; % gravity in m/s^2
L=36.0;%Pipe length in m
N_x=5;% Number of Nodes in longitudinal direction
N_y=6; % Number of Radial Nodes
cp=1000;% Specific heat of air at constant pressure in J/Kg.K
sp_ratio=1.4; % specific heat ratio
Delta_x=L/(N_x-1);% Length of each longitudinal segment in m
Delta_y=Dia/(2*(N_y-1)); % Length of each radial mesh in m
H0=4.53; % Reservoir Head in m
Area=pi*Dia^2/4;% Cross section area in m^2

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%


% PHYSICAL PROPERTIES OF AIR AND WATER AS FUNCTION OF TEMPERATURE
```

```
% Saturation vapor pressure in Pa

p_s_f=inline('610.78*exp((t/(t+238.3))*17.2694)');


% Water Density in Kg/m^3

row_w_f=inline('1000*(1-((t+288.9414)*(t-3.9863)^2/(508929.2*(t+68.12963))))');


% Air Density in Kg/m^3

row_g_f=inline('(288.16*1.2255/(273.15+T0_c))');

% Colebrook-White Formula to calculate friction factor

%% 1/sqrt(f)=-2*log(((e/D)/3.7)+(2.51/(R*sqrt(f))));

f=0.0371; % This value is obtained from above formula

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%

% INITIAL PRESSURE HEAD AND MIXTURE DENSITY CALCULATIONS

for i=1:1:N_x

   H(i,1)= H0-((8*f*(i-1)*Delta_x*Q0^2)/(pi^2*g*Dia^5))-Q0^2/((pi*Dia^2/4)^2*2*g);;

end

   % Initialising the boundary grid point near resevoir
```

```
    p(1,1)=1.418*10^5;
    row_w(1,1)=row_w_f(T0_c);
    row_g(1,1)=row_g_f(T0_c);
    row(1,1)=996.26;
    p_s(1,1)=p_s_f(T0_c);
    Ki_Visc(1,1)=.837*10^-6;

for i=2:1:N_x-1

    row_w(i,1)=row_w_f(T0_c);
    row_g(i,1)=row_g_f(T0_c);
    p_s(i,1)=p_s_f(T0_c);
    m(i,1)=(m0/(N_x-1));

    % Solve Quadratic Expression for Absolute Pressure

    a=1;                                      b=-(g*H(i,1)*row_w(i,1)+p_atm-p_s(i,1));
c0=g*m(i,1)*Gas_Const*(273.15+T0_c)*H(i,1)*(row_w(i,1)- row_g(i,1));

    p(i,1)=(-b+(b^2-4*a*c0)^.5)/(2*a);

    % Mixture Density

    row(i,1)=(p(i,1)+p_s(i,1)-p_atm)/(H(i,1)*g);


end


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%

% INITIAL VELOCITY PROFILE CALCULATIONS
```

```
% Grid points in Radial Direction

for i=1:1:N_y

    r_N(i,1)=Dia/2-(Delta_y*(N_y-i));
    if i<N_y
        r_D(i,1)=r_N(i,1)+Delta_y/2;
    else
        r_D(i,1)=r_N(i,1);

    end

end



% Reynold's Number
R=round((4*Q0)/(pi*Dia*Ki_Visc0));

% Constant K throughout simulation
k=0.374+(.0132*log(1+(83100/R)));

% Delta_A
Delta_A= (1/(N_y-1))*(pi/4)*Dia^2;



for i=2:1:N_x-1
    % Friction Velocity
    ustar(i,1)=((4*Q0)/(pi*Dia^2))*sqrt(f/8);

    % Sublayer thickness
    Delta(i,1)=11.63*Ki_Visc0/ustar(i,1);

    % Special Parameters
```

```
    Phi(i,1)=(p(i,1)/(row(i,1)*g))-
(c^2*m(i,1)*Gas_Const*(273.15+T0_c)/(g*p(i,1)))+c^2*m(i,1)/(row(i,1)*g);
    s(i,1)=cp*(log(273.15+T0_c)-(sp_ratio-1)*log(p(i,1))/sp_ratio);
end

% Validating if a Grid point is in viscous sublayer or in Transient region
% and then obtain the initial velocity profile

for j=N_y:-1:1
    u(1,N_y)=0;
end

for i=2:1:N_x-1

    for j=N_y:-1:1

        if Dia/2-r_D(j,1) <= Delta(i,1)
            u(i,j)=ustar(i,1)^2*(Dia/2-r_D(j,1))/Ki_Visc0;

        else
            u(i,j)=ustar(i,1)*(2.5*log((Dia/2-r_D(j,1))/e)+5.0);

         end
         l(j,1)=k*(Dia/2-r_N(j,1))*exp(-(Dia/2-r_N(j,1))/(Dia/2));
     end
 end




t=1;
status=1;
TOL=10^-4;
error=1;
while  status<=4
```

```
 for i=2:1:N_x-1
     count=0;
     flag=0;
     Ki_Visc(i,1)=.837*10^-6;
     for j=N_y-1:-1:1
          if Dia/2-r_N(j,1)<=Delta(i,1)

          if j==N_y-1



D(1,i)=(1/DeltaT)+(pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(N_y,1)/(r_D(N_y
,1)-r_D(N_y-1,1)))+(r_N(N_y-1,1)/(r_D(N_y-1,1)-r_D(N_y-2-1,1))));
                A(1,i)=-pi*r_N(N_y-1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(N_y-
1,1)-r_D(N_y-2,1)));
                C(1,i)=-((-u(i,N_y-1)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(N_y,1)*u(i,N_y-
1)/(r_D(N_y,1)-r_D(N_y-1,1)))-(r_N(N_y-1,1)*u(i,N_y-2)/(r_D(N_y-1,1)-r_D(N_y-
2,1)))+(r_N(N_y-1,1)*u(i,N_y-1)/(r_D(N_y-1,1)-r_D(N_y-2,1)))));
                flag=flag+1;
            else
                B(2+count,i)=-
pi*Ki_Visc(i,t)*row_w(i,t)*r_N(j+1,1)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1)));

D(2+count,i)=(1/DeltaT)+(pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_
D(j+1,1)-r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-1,1))));
                A(2+count,i)=-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-
r_D(j-1,1)));
                C(2+count,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(j+1,1)*(u(i,j)-
u(i,j+1))/(r_D(j+1,1)-r_D(j,1)))-(r_N(j,1)*(u(i,j-1)-u(i,j))/(r_D(j,1)-r_D(j-1,1)))));
                count=count+1;
                flag=flag+1;
            end
```

```
        end
            if Dia/2-r_N(j,1)>Delta(i,1)

              if j==N_y-1



D(1,i)=((1/DeltaT)+(pi*Ki_Visc(i,t)*row_w(i,t)*r_N(j+1,1))/(row(i,t)*Delta_A*(r_D(j+1,1)-
r_D(j,1))))+((pi*r_N(j,1)*row_w(i,t))/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
              A(1,i)=-(pi*r_N(j,1)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
              C(1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*(r_N(j+1,1)/r_D(j+1,1)-
r_D(j,1))*u(i,j)+(pi*r_N(j,1)*row_w(i,t)*(u(i,j)-u(i,j-1))/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))));
              flag=flag+1;

              elseif Dia/2-r_N(j+1,1)< Delta(i,1)

              B(flag+1,i)=                                                          -
pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1)));
              D(flag+1,i)=
(1/DeltaT)+((pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_D(j+1,1)-
r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-
1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-
1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
              A(flag+1,i)=    (-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-
r_D(j-1,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-
1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
              C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+((pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(j+1,1)*(u(i,j)-
u(i,j+1))/(r_D(j+1,1)-r_D(j,1)))+(r_N(j,1)*(u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-
1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-
r_D(j-1,1)))*((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))));
```

```
flag=flag+1;

elseif r_N(j,1) ~= 0

B(flag+1,i)=(-pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-
r_D(j,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-
r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
D(flag+1,i)=(1/DeltaT)+((pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_D
(j+1,1)-r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-
1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*((l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-r_D(j-
1,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))))+(l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-
1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))))));
A(flag+1,i)=(-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-1,1))))-
((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-1,1)))*abs((u(i,j)-
u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-H(i,1))/Delta_x)-
((pi/row(i,t))*(r_N(j+1,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-
r_D(j,1)))+row_w(i,t)*l(j+1,1)^2*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))*((u(i,j+1)-
u(i,j))/(r_D(j+1,1)-
r_D(j,1)))))+((pi/row(i,t))*(r_N(j,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j)-u(i,j-
1))/(r_D(j,1)-r_D(j-1,1)))+row_w(i,t)*l(j,1)^2*abs((u(i,j)-u(i,j))/(r_D(j,1)-r_D(j-1,1)))*((u(i,j)-
u(i,j-1))/(r_D(j,1)-r_D(j-1,1))))));
flag=flag+1;
elseif r_N(j,1) == 0

D(flag+1,i)=(1/DeltaT)+(pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1
)-r_D(j,1))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-
r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
B(flag+1,i)=(-pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-
r_D(j,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-
r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-H(i,1))/Delta_x)-
((pi/row(i,t))*(r_N(j+1,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-
```

```
r_D(j,1)))+row_w(i,t)*l(j+1,1)^2*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))*((u(i,j+1)-
u(i,j))/(r_D(j+1,1)-r_D(j,1))))));

end
end
end
end

% Thomas Algorithm

B(1,1)=0;
for i=2:1:N_x-1
D1(1,i)=D(1,i);
C1(1,i)=C(1,i);
B(1,i)=0;

for j=2:1:N_y-1

    D1(j,i)=D(j,i)-(B(j,i)*A(j-1,i)/D1(j-1,i));
    C1(j,i)=C(j,i)-(C1(j-1,i)*B(j,i)/D1(j-1,i));
end

  u(i,1)=C1(N_y-1,i)/D1(N_y-1,i);

  for j=2:1:N_y-1
  u(i,j)=(C1(N_y-j,i)-A(N_y-j,i)*u(i,j-1))/D1(N_y-j,i);

end
end
status=status+1;
end

% OBTAIN DISCHARGE AT A SECTION FROM VELOCITY PROFILE:
```

```
for i=2:1:N_x-1
temp(i,1)=u(i,1)*pi*r_N(2,1)^2;
tempo=0;
for j=2:1:N_y-1
tempo=tempo+u(i,j)*pi*(r_N(j+1,1)^2-r_N(j,1)^2);
end
Q(i,1)=temp(i,1)+tempo;
end

Q(1,1)=2*Q(2,1)-Q(3,1);
Q(N_x,1)=0;
m(N_x,1)=m0/(N_x-1);


% BEGIN OF MACCORMACK SCHEME

%Setting up the initial guess values

for i=2:1:N_x-1
   p_p(i,1)=p(i,1);
   p_c(i,1)=p(i,1);

end


for t=1:1:40

% Predictor Step
   t

   for i=2:1:N_x-1
      T(i,1)=T0_k;
      Phi_p(i,1)=Phi(i,t)+(c^2/(g*Area))*(Q(i,t)-Q(i+1,t))*(DeltaT/Delta_x);
      s_p(i,1)=s(i,t)+(cp/Theta_T)*((T0_k/T(i,t))-1)*DeltaT;
```

```
        m_p(i,1)=m(i,t)+0.02*(DeltaT/(Theta_m*Gas_Const*T(i,t)))*(p_s(i,t)-p(i,t));

        count=0;
        flag=0;
        Ki_Visc(i,t)=.837*10^-6;



        for j=N_y-1:-1:1
           if Dia/2-r_N(j,1)<=Delta(i,1)

              if j==N_y-1


D(1,i)=(1/DeltaT)+(pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(N_y,1)/(r_D(N_y
,1)-r_D(N_y-1,1)))+(r_N(N_y-1,1)/(r_D(N_y-1,1)-r_D(N_y-2-1,1))));
                 A(1,i)=-pi*r_N(N_y-1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(N_y-
1,1)-r_D(N_y-2,1)));
                 C(1,i)=-((-u(i,N_y-1)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(N_y,1)*u(i,N_y-
1)/(r_D(N_y,1)-r_D(N_y-1,1)))-(r_N(N_y-1,1)*u(i,N_y-2)/(r_D(N_y-1,1)-r_D(N_y-
2,1)))+(r_N(N_y-1,1)*u(i,N_y-1)/(r_D(N_y-1,1)-r_D(N_y-2,1)))));
                 flag=flag+1;
              else
                 B(2+count,i)=-
pi*Ki_Visc(i,t)*row_w(i,t)*r_N(j+1,1)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1)));

D(2+count,i)=(1/DeltaT)+(pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_
D(j+1,1)-r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-1,1))));
                 A(2+count,i)=-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-
r_D(j-1,1)));
                 C(2+count,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(j+1,1)*(u(i,j)-
u(i,j+1))/(r_D(j+1,1)-r_D(j,1)))-(r_N(j,1)*(u(i,j-1)-u(i,j))/(r_D(j,1)-r_D(j-1,1)))));
```

```
            count=count+1;
            flag=flag+1;
          end
        end
        if Dia/2-r_N(j,1)>Delta(i,1)

          if j==N_y-1



D(1,i)=((1/DeltaT)+(pi*Ki_Visc(i,t)*row_w(i,t)*r_N(j+1,1))/(row(i,t)*Delta_A*(r_D(j+1,1)-
r_D(j,1))))+((pi*r_N(j,1)*row_w(i,t))/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
            A(1,i)=-(pi*r_N(j,1)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
            C(1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*(r_N(j+1,1)/r_D(j+1,1)-
r_D(j,1))*u(i,j)+(pi*r_N(j,1)*row_w(i,t)*(u(i,j)-u(i,j-1))/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))));
            flag=flag+1;

          elseif Dia/2-r_N(j+1,1)< Delta(i,1)


            B(flag+1,i)=                                                    -
pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1)));
            D(flag+1,i)=
(1/DeltaT)+((pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_D(j+1,1)-
r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-
1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-
1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
            A(flag+1,i)=   (-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-
r_D(j-1,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-
1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
            C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-
H(i,1))/Delta_x)+((pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(j+1,1)*(u(i,j)-
```

```
u(i,j+1))/(r_D(j+1,1)-r_D(j,1)))+(r_N(j,1)*(u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))*((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))));
          flag=flag+1;

        elseif r_N(j,1) ~= 0

          B(flag+1,i)=(-pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));

D(flag+1,i)=(1/DeltaT)+((pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_D(j+1,1)-r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*((l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))))+(l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))))));
          A(flag+1,i)=(-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-1,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
          C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-H(i,1))/Delta_x)-((pi/row(i,t))*(r_N(j+1,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))+row_w(i,t)*l(j+1,1)^2*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))))+((pi/row(i,t))*(r_N(j,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))+row_w(i,t)*l(j,1)^2*abs((u(i,j)-u(i,j))/(r_D(j,1)-r_D(j-1,1)))*((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))))));
          flag=flag+1;
        elseif r_N(j,1) == 0

D(flag+1,i)=(1/DeltaT)+(pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
```

```
                B(flag+1,i)=(-
pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1))))-
((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-
r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
                C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H(i+1,1)-H(i,1))/Delta_x)-
((pi/row(i,t))*(r_N(j+1,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-
r_D(j,1)))+row_w(i,t)*l(j+1,1)^2*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))*((u(i,j+1)-
u(i,j))/(r_D(j+1,1)-r_D(j,1))))));

            end
        end
    end
end




% Thomas Algorithm

B(1,1)=0;
for i=2:1:N_x-1
    D1(1,i)=D(1,i);
    C1(1,i)=C(1,i);
    B(1,i)=0;

    for j=2:1:N_y-1

        D1(j,i)=D(j,i)-(B(j,i)*A(j-1,i)/D1(j-1,i));
        C1(j,i)=C(j,i)-(C1(j-1,i)*B(j,i)/D1(j-1,i));
    end

    u_p(i,1)=C1(N_y-1,i)/D1(N_y-1,i);

    for j=2:1:N_y-1
        u_p(i,j)=(C1(N_y-j,i)-A(N_y-j,i)*u(i,j-1))/D1(N_y-j,i);
```

```
    end
end

% Finding predictor value of Discharge Q
for i=2:1:N_x-1
    temp(i,1)=u_p(i,1)*pi*r_N(2,1)^2;
    tempo=0;
    for j=2:1:N_y-1
        tempo=tempo+u_p(i,j)*pi*(r_N(j+1,1)^2-r_N(j,1)^2);
    end
    Q_p(i,1)=temp(i,1)+tempo;
end



%  Finding predictor value of Pressure p and Temperature T

for i=2:1:N_x-1

    cff1=(1/(row(i,t)*g));
    cff2=-(c^2*m_p(i,1)*Gas_Const/g);
    cff3=((c^2*m_p(i,1)/(row(i,t)*g))-Phi_p(i,1));
    cff4=(s_p(i,1)/cp);
    cff5=((sp_ratio-1)/sp_ratio);

    f01=inline('cf1*x^2+cf2*exp(cf5*log(x)+cf4)+cf3*x','x','cf1','cf2','cf3','cf4','cf5');
    [p_p(i,1),fval,exitFlag]=fzero(f01,p_p(i,1),[],cff1,cff2,cff3,cff4,cff5);

    if(exitFlag<0)
     %   fprintf('You faced a NaN value')
     p_p(i,1)=fzero(f01,100,[],cff1,cff2,cff3,cff4,cff5);
    end
    T_p(i,1)=exp(cff5*log(p_p(i,1))+cff4);
```

```
    if T_p(i,1)<273.15
        p_s_p(i,1)=0;
    else
        p_s_p(i,1)=p_s_f(T_p(i,1)-273.15);
    end

    H_p(i,1)=(p_p(i,1)+p_s_p(i,1)-p_atm)/(row(i,t)*g);
end

%Interpolating to obtain the values at Boundary points
H_p(1,1)=H0;
Q_p(1,1)=2*Q_p(2,1)-Q_p(3,1);



% Corrector Step
for i=2:1:N_x-1

    Phi_c(i,1)=Phi(i,t)+(c^2/(g*Area))*(Q_p(i-1,1)-Q_p(i,1))*(DeltaT/Delta_x);
    s_c(i,1)=s(i,t)+(cp/Theta_T)*((T0_k/T_p(i,1))-1)*DeltaT;
    m_c(i,1)=m(i,t)+0.02*(DeltaT/(Theta_m*Gas_Const*T(i,t)))*(p_s_p(i,1)-p_p(i,1));

    count=0;
    flag=0;
    Ki_Visc(i,t)=.837*10^-6;



    for j=N_y-1:-1:1
        if Dia/2-r_N(j,1)<=Delta(i,1)

            if j==N_y-1
```

```
D(1,i)=(1/DeltaT)+(pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(N_y,1)/(r_D(N_y
,1)-r_D(N_y-1,1)))+(r_N(N_y-1,1)/(r_D(N_y-1,1)-r_D(N_y-2-1,1))));
          A(1,i)=-pi*r_N(N_y-1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(N_y-
1,1)-r_D(N_y-2,1)));
          C(1,i)=-((-u(i,N_y-1)/DeltaT)+(g*(H_p(i,1)-H_p(i-
1,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(N_y,1)*u(i,N_y-
1)/(r_D(N_y,1)-r_D(N_y-1,1)))-(r_N(N_y-1,1)*u(i,N_y-2)/(r_D(N_y-1,1)-r_D(N_y-
2,1)))+(r_N(N_y-1,1)*u(i,N_y-1)/(r_D(N_y-1,1)-r_D(N_y-2,1)))));
          flag=flag+1;
        else
          B(2+count,i)=-
pi*Ki_Visc(i,t)*row_w(i,t)*r_N(j+1,1)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1)));

D(2+count,i)=(1/DeltaT)+(pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_
D(j+1,1)-r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-1,1))));
          A(2+count,i)=-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-
r_D(j-1,1)));
          C(2+count,i)=-((-u(i,j)/DeltaT)+(g*(H_p(i,1)-H_p(i-
1,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(j+1,1)*(u(i,j)-
u(i,j+1))/(r_D(j+1,1)-r_D(j,1)))-(r_N(j,1)*(u(i,j-1)-u(i,j))/(r_D(j,1)-r_D(j-1,1)))));
          count=count+1;
          flag=flag+1;
        end
      end
      if Dia/2-r_N(j,1)>Delta(i,1)

        if j==N_y-1


D(1,i)=((1/DeltaT)+(pi*Ki_Visc(i,t)*row_w(i,t)*r_N(j+1,1))/(row(i,t)*Delta_A*(r_D(j+1,1)-
r_D(j,1))))+((pi*r_N(j,1)*row_w(i,t))/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
```

```
            A(1,i)=-(pi*r_N(j,1)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
            C(1,i)=-((-u(i,j)/DeltaT)+(g*(H_p(i,1)-H_p(i-
1,1))/Delta_x)+(pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*(r_N(j+1,1)/r_D(j+1,1)-
r_D(j,1))*u(i,j)+(pi*r_N(j,1)*row_w(i,t)*(u(i,j)-u(i,j-1))/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-
1,1))))*(Ki_Visc(i,t)+l(j,1)^2*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))));
            flag=flag+1;

        elseif Dia/2-r_N(j+1,1)< Delta(i,1)

            B(flag+1,i)=                                                    -
pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1)));
            D(flag+1,i)=
(1/DeltaT)+((pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_D(j+1,1)-
r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-
1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-
1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
            A(flag+1,i)=    (-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-
r_D(j-1,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-
1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
            C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H_p(i,1)-H_p(i-
1,1))/Delta_x)+((pi/row(i,t))*(Ki_Visc(i,t)*row_w(i,t)/Delta_A)*((r_N(j+1,1)*(u(i,j)-
u(i,j+1))/(r_D(j+1,1)-r_D(j,1)))+(r_N(j,1)*(u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-
1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-
r_D(j-1,1)))*((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))));
            flag=flag+1;

        elseif r_N(j,1) ~= 0

            B(flag+1,i)=(-
pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1))))-
((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-
r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
```

```
D(flag+1,i)=(1/DeltaT)+((pi/row(i,t))*(1/Delta_A)*Ki_Visc(i,t)*row_w(i,t)*((r_N(j+1,1)/(r_D(j+1,1)-r_D(j,1)))+(r_N(j,1)/(r_D(j,1)-r_D(j-1,1)))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*((l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-r_D(j-1,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))))+(l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))))));
          A(flag+1,i)=(-pi*r_N(j,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j,1)-r_D(j-1,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j,1)^2*r_N(j,1)*(1/(r_D(j,1)-r_D(j-1,1)))*abs((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))));
          C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H_p(i,1)-H_p(i-1,1))/Delta_x)-((pi/row(i,t))*(r_N(j+1,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))+row_w(i,t)*l(j+1,1)^2*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))))+((pi/row(i,t))*(r_N(j,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1)))+row_w(i,t)*l(j,1)^2*abs((u(i,j)-u(i,j))/(r_D(j,1)-r_D(j-1,1)))*((u(i,j)-u(i,j-1))/(r_D(j,1)-r_D(j-1,1))))));
          flag=flag+1;
        elseif r_N(j,1) == 0

D(flag+1,i)=(1/DeltaT)+(pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1))))+((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
          B(flag+1,i)=(-pi*r_N(j+1,1)*Ki_Visc(i,t)*row_w(i,t)/(row(i,t)*Delta_A*(r_D(j+1,1)-r_D(j,1))))-((pi/row(i,t))*(1/Delta_A)*row_w(i,t)*l(j+1,1)^2*r_N(j+1,1)*(1/(r_D(j+1,1)-r_D(j,1)))*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))));
          C(flag+1,i)=-((-u(i,j)/DeltaT)+(g*(H_p(i,1)-H_p(i-1,1))/Delta_x)-((pi/row(i,t))*(r_N(j+1,1)/Delta_A)*(Ki_Visc(i,t)*row_w(i,t)*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))+row_w(i,t)*l(j+1,1)^2*abs((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1)))*((u(i,j+1)-u(i,j))/(r_D(j+1,1)-r_D(j,1))))));

        end
      end
```

```
    end
end



% Thomas Algorithm

B(1,1)=0;
for i=2:1:N_x-1
    D1(1,i)=D(1,i);
    C1(1,i)=C(1,i);
    B(1,i)=0;

    for j=2:1:N_y-1

        D1(j,i)=D(j,i)-(B(j,i)*A(j-1,i)/D1(j-1,i));
        C1(j,i)=C(j,i)-(C1(j-1,i)*B(j,i)/D1(j-1,i));
    end

    u_c(i,1)=C1(N_y-1,i)/D1(N_y-1,i);

    for j=2:1:N_y-1
        u_c(i,j)=(C1(N_y-j,i)-A(N_y-j,i)*u(i,j-1))/D1(N_y-j,i);

    end
end

% Finding predictor value of Discharge Q
for i=2:1:N_x-1
    temp(i,1)=u_c(i,1)*pi*r_N(2,1)^2;
    tempo=0;
    for j=2:1:N_y-1
        tempo=tempo+u_c(i,j)*pi*(r_N(j+1,1)^2-r_N(j,1)^2);
    end
```

```
    Q_c(i,1)=temp(i,1)+tempo;
end
%  Finding corrector value of Pressure p and Temperature T

for i=2:1:N_x-1

    cff1=(1/(row(i,t)*g));
    cff2=-(c^2*m_c(i,1)*Gas_Const/g);
    cff3=((c^2*m_c(i,1)/(row(i,t)*g))-Phi_c(i,1));
    cff4=(s_c(i,1)/cp);
    cff5=((sp_ratio-1)/sp_ratio);

    f01=inline('cf1*x^2+cf2*exp(cf5*log(x)+cf4)+cf3*x','x','cf1','cf2','cf3','cf4','cf5');
    [p_c(i,1),fval,exitFlag]=fzero(f01,p_c(i,1),[],cff1,cff2,cff3,cff4,cff5);

    if(exitFlag<0)
     %   fprintf('You faced a NaN value')
     p_c(i,1)=fzero(f01,100,[],cff1,cff2,cff3,cff4,cff5);
    end
    T_c(i,1)=exp(cff5*log(p_c(i,1))+cff4);

    if T_c(i,1)<273.15
        p_s_c(i,1)=0;
    else
        p_s_c(i,1)=p_s_f(T_c(i,1)-273.15);
    end

    H_c(i,1)=(p_c(i,1)+p_s_c(i,1)-p_atm)/(row(i,t)*g);
end

%Interpolating to obtain the values at Boundary points
H_c(1,1)=H0;
Q_c(1,1)=2*Q_c(2,1)-Q_c(3,1);
```

```
% OBTAINING THE PARAMETERS

for i=2:1:N_x-1

    Phi(i,t+1)=0.5*(Phi_p(i,1)+Phi_c(i,1));
    s(i,t+1)=0.5*(s_p(i,1)+s_c(i,1));
    m(i,t+1)=0.5*(m_p(i,1)+m_c(i,1));
    Q(i,t+1)=0.5*(Q_p(i,1)+Q_c(i,1));
    p(i,t+1)=0.5*(p_p(i,1)+p_c(i,1));
    T(i,t+1)=0.5*(T_p(i,1)+T_c(i,1));
    p_s(i,t+1)=0.5*(p_s_p(i,1)+p_s_c(i,1));
    H(i,t+1)=0.5*(H_p(i,1)+H_c(i,1));
    for j=2:1:N_y-1
        u(i,j)=0.5*(u_p(i,j)+u_c(i,j));
    end

    row_w(i,t+1)=row_w_f(T(i,t+1)-273.15);
    row_g(i,t+1)=row_g_f(T(i,t+1)-273.15);
end

Q(1,t+1)=2*Q(2,t+1)-Q(3,t+1);
H(1,t+1)=4.53;
H(N_x,t+1)=2*H(N_x-1,t+1)-H(N_x-2,t+1);
T(N_x,t+1)=2*T(N_x-1,t+1)-T(N_x-2,t+1);
m(N_x,t+1)=2*m(N_x-1,t+1)-m(N_x-2,t+1);

%Find Mixture Density
for i=2:1:N_x-1
%                                                              row(i,t+1)=(1-
m(i,t+1)*Gas_Const*T(i,t+1)/p(i,t+1))*row_w(i,t+1)+(m(i,t+1)*Gas_Const*T(i,t+1)/p(i,t+1))*
row_g(i,t+1);
row(i,t+1)=row(i,1);
end
```

```
    %Find Mass of gas released
    hold=0;
    for i=2:1:N_x
        hold=hold+m(i,t);
    end
    mass(t,1)=hold;

end



%% THE GRAPHS AND RESULTS
figure(1)
t=[1:1:40]';

plot(t,H(5,[1:1:40]));
grid on;
xlabel('t');
ylabel('H(m)');
title('Head Near the Valve');

figure(2)
plot(t,T(5,[1:1:40])-273.15);
grid on;
xlabel('t');
ylabel('T(deg c)');
title('Temperature Near the Valve');

figure(3)
plot(t,mass([1:1:40],1)*10^6);
grid on;
xlabel('t');
ylabel('m(mg/m3)');
title('Mass of gas release');
```

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