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Particle size effects on optimal sizing and lifetime of pipelines transporting multi-sized solid-liquid mixtures

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Abstract

A life-cycle cost analysis model is developed in this study, to examine the effects of particle size distribution of the solid particles to be transported on the optimal sizing and lifetime of the pipelines used for transportation of solid-liquid mixtures. The method determines the lifetime of the pipe corresponding to the least annual total cost per unit length of the pipe. The optimum diameter is obtained so that the total cost per unit pipe length per unit volume of the transported mixture throughout this lifetime is minimum. The total cost includes manufacturing and repair cost of pipe, cost of pumping power as well as the cost of power required for the crushing of particles from an initial size distribution to a desirable particle size distribution. The repair cost of pipe and cost of pumping power increase as the pipe becomes older due to more frequent pipe breaks and due to the pipe wear that makes wall roughness, and thereby pressure drop, greater. These costs together with the cost of power for crushing must be considered for through life costing of pipelines. Since the transportation of solid-liquid mixtures is maintained by several pumping stations in long pipelines, the spacing between two successive pumping stations must also be determined. The study shows interdependence of parameters such as the lifetime, the optimum diameter, the corresponding spacing for a given pumping power and the particle size distribution of solid particles transported in the pipeline. Furthermore, the method also provides the interrelation between the total length of pipeline when crushing is economical and the different particle size distributions.

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1. Introduction

A cost effective and environmentally friendly method of transporting minerals in mining industries is by pumping the extracted solid material in crushed form along with a carrier liquid in a pipeline. During the life cycle of such pipelines, several cost elements contribute to the total cost. These elements include manufacturing and repair costs, cost of pumping power as well as cost of power required for crushing of particles. These cost elements are determined by various geometric, fluid and flow parameters, but some of them also vary considerably with time. Therefore, in order to obtain a pipe size that is associated with the minimum total cost, a life-cycle analysis should be carried out instead of the conventional optimization procedure.

An important factor that is normally considered as a constant in the optimization procedure, or not considered at all, is the effect associated with the size of solid particles. The particle size of the carried solids may vary by three orders of magnitude and has a considerable effect on the optimum diameter of pipeline. Furthermore, since it is a critical factor influencing pipe wear, the lifetime of pipeline also depends significantly on the size of solid particles being transported.

A method for optimal sizing of capsule handling multi-phase pipelines was proposed in [1]. The particle size effect on pressure drop in pipelines carrying multi-sized slurries was studied in [2]. A life-cycle energy analysis quantifying the energy expenditures in different life stages of pipes of a water distribution system was presented in [3]. This idea is applied

in the present study to develop a life-cycle cost model for pipelines transporting multi-sized solid-liquid mixtures.

The goal of this study is to investigate the dependence of optimum diameter and lifetime of pipelines transporting solid-liquid mixtures on the particle size of the transported solid. Moreover, the spacing between two successive pumping stations will also be determined for pipelines transporting different sizes of solid particles when the power of pumping unit is known. In order to achieve these goals, a life-cycle cost analysis model is developed where the particle size effects are incorporated by the weighted mean diameter. The weighted mean diameter is calculated for different particle size distributions that consider fine and coarse slurries and that are defined by the Rosin-Rammler distribution. In the following, first the total cost model will be described, and then the results obtained from the analysis will be presented.

2. Modelling

The life-cycle cost analysis of pipelines transporting solid-liquid mixtures determines a lifetime that corresponds to the least annual total cost per unit length of the pipe. Then, minimizing the total cost per unit pipe length per unit volume of transported mixture throughout this lifetime provides the pipeline diameter. This section presents the cost model.

2.1. Pumping energy

In order to calculate the cost of pumping energy, the head loss in the pipeline should be predicted. The pressure drop in a flow of a solid-liquid mixture is different from that in a flow of a single liquid. In the equation of head loss, Durand [4] proposed a term that considers the effects of solid particles. Neglecting minor losses, the head loss in the pipeline can be calculated from Equation 1.

$$\Delta h_s = \frac{fV^2}{2gD} + \frac{K\sqrt{gD}(S-1)^{1.5}C_v f}{VC_D^{0.75}} \quad (1)$$

In this equation, f is the friction factor, V is the flow velocity, g is the acceleration due to gravity, D is the pipe diameter, S is specific gravity, C_v is the volume fraction of solids in the mixture, C_D is the drag coefficient of the solid particles, and Durand [4] proposed that K is a constant. The friction factor f can be obtained from Wood's equation [5]:

$$f = 0.53 \frac{\varepsilon}{D} + 0.094 \left(\frac{\varepsilon}{D} \right)^{0.225} + 88 \left(\frac{\varepsilon}{D} \right)^{0.44} \text{Re}^{-1.62} \left(\frac{\varepsilon}{D} \right)^{0.134} \quad (2)$$

where ε is pipe roughness, $\text{Re} = \rho_L VD/\mu$ is Reynolds number, and ρ_L and μ are density and dynamic viscosity of the carrying liquid, respectively. The velocity of flow is fixed in the present model so that its value is slightly higher than the deposition velocity, V_D : $V = V_D + 0.2$. The deposition velocity is calculated from Wick's equation [6]:

$$V_D = 1.87 \left(\frac{d}{D} \right)^{1/6} \left[\frac{2gD(\rho_s - \rho_L)}{\rho_L} \right]^{0.5} \quad (3)$$

with ρ_s denoting solid density. The volume fraction of solid C_v can be calculated from the known solid throughput Q_s :

$$Q_s = \frac{\pi D^2 V C_v \rho_s}{4} \quad (4)$$

The total volumetric flow rate Q in the pipeline is the sum of the flow rates of the carrier fluid and the solid phase. If no slip is assumed, then it is obtained from Equation 5.

$$Q = \frac{Q_s}{\rho_s} + \frac{\pi D^2 V}{4} (1 - C_v) \quad (5)$$

The drag coefficient C_D is obtained from the slurry Reynolds number Re_s that is a function of settling velocity V_0 , particle diameter d , liquid density ρ_L , and liquid viscosity μ

$$C_D = \frac{24}{\text{Re}_s} \left(1 + \frac{3}{16} \text{Re}_s \right) \quad (6)$$

The settling velocity V_0 of the solid particles is obtained depending on particle size as follows:

$$V_0 = \frac{g(\rho_s - \rho_L)d^2}{18\mu_L} \quad \text{if } d < \left(\frac{3.6\mu_L}{[g(\rho_s - \rho_L)]^{0.28} \rho_L^{0.27}} \right)^{\frac{1}{0.82}}$$

$$V_0 = \frac{0.2 \left[\frac{g(\rho_s - \rho_L)}{\rho_L} \right]^{0.72} d^{1.18}}{\left(\frac{\mu_L}{\rho_L} \right)^{0.45}} \quad \text{otherwise} \quad (7)$$

The constant K in Equation 1 is modified so that it incorporates particle size distribution effects:

$$K = A \left(\frac{V}{V_0} \right)^a \left(\frac{C_v}{C_{vss}} \right)^b \left(\frac{d}{D} \right)^c \quad (8)$$

The parameter C_{vss} is the final static settled concentration that depends on particle diameter according to Equation 9 [7]

$$C_{vss} = B_0 + B_1 \log d \quad (9)$$

The parameters B_0 and B_1 were obtained for different materials in [7]. The constants $A = 0.005$, $a = -0.2$, $b = -0.9$ and $c = -1.2$ were determined by fitting on data measured in [2] and [7].

The efficiency η consists of two parts: pump's efficiency that depends on suspension properties and motor drive efficiency η_m that does not depend on them: $\eta = \eta_s \eta_m =$

$ER\eta_w\eta_m$. The effect of solid on pump's efficiency is considered by the efficiency ratio ER that is the ratio of efficiency of the pump for solid η_s to the efficiency of the pump for water η_w .

2.2. Particle size distribution

The size distribution of solid particles is modelled through the Rosin-Rammler distribution. The fraction of the total volume F contained in particles of diameter less than diameter d^* is expressed in the form [8]:

$$F = 1 - \exp\left[-\left(\frac{d^*}{X}\right)^q\right] \tag{10}$$

The parameter X in this distribution is the particle diameter such that 63.2% of the total volume of particles is of smaller in size as compared to this size. The second parameter q is a measure of the spread of particle sizes. Particles are represented by the weighted mean diameter in the life-cycle cost analysis, which is calculated by collecting all the particles in N bins and using the following formula

$$d = \frac{\sum_{i=1}^N w_i d_i}{\sum_{i=1}^N w_i} \tag{11}$$

In this equation, d_i is the average diameter of particles in the i^{th} bin, and $w_i = F_i - F_{i-1}$ (with $F_0 = 0$) is the fraction of weight of particles in the i^{th} bin.

2.3. Pipe wear

The head loss that the pumping energy must overcome depends on the roughness of pipe wall. This parameter increases with the lifetime of the pipe. Thus, its time dependence may be considered by defining an initial roughness height ε_0 and a roughness growth rate E_w . The initial roughness height may be taken from data available for commercial pipes, whereas the roughness growth rate may be estimated using Equation 12 [7].

$$E_w = 0.178 V^{2.4882} d^{0.291} C_w^{0.516} \tag{12}$$

where C_w is the mass fraction of solid, and flow velocity and particle diameter are substituted in m/s and mm, respectively. Although the parameters in the above equation may vary in the optimization procedure, the roughness growth rate is assumed to be constant in the analysis for a given particle size distribution. A further simplification in the model is that the pipe roughness takes an average constant value in each 5-year interval.

2.4. Crushing

When the solid particles are coarse, it may be economical to apply a size reduction method. The present model considers crushing of particles. The energy requirement for this process E_{crush} may be obtained from the Bond crushing law for particles in the range of a few 10 μm to a few 10 mm as follows [9,10]

$$E_{crush} = \frac{P_{crush}}{\dot{m}} = 10W_i \left(\frac{1}{\sqrt{d_{80,f}}} - \frac{1}{\sqrt{d_{80,in}}} \right) \tag{13}$$

Here, the parameter W_i is the work index that is the energy required in kWh per ton of feed to reduce a very large feed to such a size that 80% of the product passes a 100 μm screen. The initial and final particle diameters $d_{80,in}$ and $d_{80,f}$ describe the particle size distributions so that 80% of the total volume of particles has smaller size, and they are substituted in μm . The feed rate \dot{m} is obtained from the solid throughput Q_S , and it is substituted in ton/h. Then, the required power P_{crush} is obtained in kW.

2.5. Cost model

The total cost of the piping system includes the cost of pumping energy, cost of pipe and cost of crushing. The power required per unit pipe length P_L to transport a unit volume of mixture is calculated from the total head loss in the pipeline as follows:

$$\frac{P_L}{Q} = \frac{\rho_L g \Delta h_S}{\eta} \tag{14}$$

Then, the levelized net cost of pumping energy per unit pipe length per unit volume of transported mixture throughout the lifetime of the pipeline is computed from Equation 15.

$$C_{power} = \frac{\rho_L g \sum_{i=1}^{t_T/\Delta t} C_{1,i} \Delta h_{S,i} \Delta t}{\eta} \tag{15}$$

The symbols t_T and Δt denote the lifetime of the pipeline and the 5-year interval when the roughness is assumed to be constant, respectively. The parameter $C_{1,i}$ is the levelized net annual cost of the energy consumed per unit power used by the pumping unit installed. This parameter is also assumed to be constant in each 5-year interval.

The cost of pipe is the sum of manufacturing cost and repair cost. The levelized net manufacturing cost of pipe per unit pipe length per unit volume of transported mixture is obtained from Equation 16.

$$C_{manuf} = \frac{4C_2 C_c \gamma_p}{V} \tag{16}$$

where C_2 is the levelized net cost of pipe per unit weight of pipe material, C_c provides relationship between pipe wall

thickness t and pipe diameter D : $t = C_c D$, and γ_p stands for specific weight for pipe material. The cost of repair is calculated from Equation 17 [3].

$$C_{repair} = \frac{C_{break}}{Q} \sum_{i=t_1}^{t_n} N(i) \tag{17}$$

Here, C_{break} is the cost of repairing a single break, which is calculated by multiplying a typical break length L_b with the manufacturing cost C_{manuf} . A typical pipe-break length recommended in the literature $L_b = 9$ m/break (see [3] and references therein) will be used in the present analysis. The other term in the above equation is the sum of break rates in each year. The break rate is determined as follows

$$N(t_i) = N(t_1) e^{\phi(t_i - t_1)} \tag{18}$$

where t_1 is initial time (or time of pipe replacement), $N(t_1)$ is initial break rate, and ϕ is breakage growth rate. Typical values for initial break rate and breakage growth rate are 0.04 breaks/km/year and 0.07/year, respectively [3]. The present model takes these values.

The cost of power required for crushing per unit pipe length per unit volume of transported mixture throughout the lifetime of the pipeline is obtained as follows

$$C_{crush} = \frac{P_{crush}}{Q L_{total}} \sum_{i=1}^{t_r/\Delta t} C_{1,i} \Delta t \tag{19}$$

It is assumed that crushing is maintained throughout the lifetime of the pipeline; therefore, the power P_{crush} is multiplied by the time dependent annual cost of the energy consumed per unit power $C_{1,i}$ and the corresponding time interval Δt . Furthermore, crushing is applied at the site of mining process where the transportation begins. Thus, the cost of crushing is independent of pipe length, or it may be assumed that it is related to the total length of pipeline. Since the present model considers costs for a unit pipe length, the cost of crushing is divided by the total length L_{total} .

Finally, the levelized total cost of the pipeline per unit length per unit volume of transported mixture throughout the lifetime of the pipeline is the sum of the cost of pumping energy, the cost of pipes and the cost of crushing, as shown in Equation 20.

$$C_{total} = C_{power} + C_{manuf} + C_{repair} + C_{crush} \tag{20}$$

2.6. Pipe length

The length of pipe between two successive pumping units, L , can be determined if the pumping power, P , is known beforehand. The pumping power per unit pipe length is calculated as follows:

$$P_L = \frac{\rho_L g Q \sum_{i=1}^{t_r/\Delta t} \Delta h_{s,i} \Delta t}{\eta T_r} \tag{21}$$

Then, if the pumping power is known, the pipe length is obtained from Equation 22.

$$L = \frac{P}{P_L} \tag{22}$$

3. Particle size effects

The life-cycle cost analysis is carried out for varying particle size distributions of the solid transported in the pipe. Six different particle size distributions are determined using the Rosin-Rammler distribution. The distributions as defined by the parameters X and q in Table 1 represent fine, mid-size and coarse solids with particle sizes spread in a narrow or wide region. The same table also includes the corresponding weighted mean diameter, the diameter so that 95% of the total volume of particles has smaller size, which provides approximate information about the greatest particles in the distribution, and the roughness growth rate that increases when the solid is coarser. Fig. 1 shows the six cumulative particle size distributions.

Table 1. Solid with different particle size distributions and values of relevant parameters (X, q are parameters in Rosin-Rammler distribution, d – weighted mean diameter, d_{95} – diameter so that 95% of the total volume of particles has smaller size, E_w – roughness growth rate)

Solid / size region	X (μm)	q	d (μm)	d_{95} (μm)	E_w (mm/yr)
Fine / narrow	25	5	23	31	0.13
Fine / wide	25	0.5	50	225	0.17
Mid-size / narrow	100	5	92	125	0.2
Mid-size / wide	100	0.5	200	900	0.25
Coarse / narrow	500	5	460	620	0.3
Coarse / wide	500	0.5	1000	4500	0.4

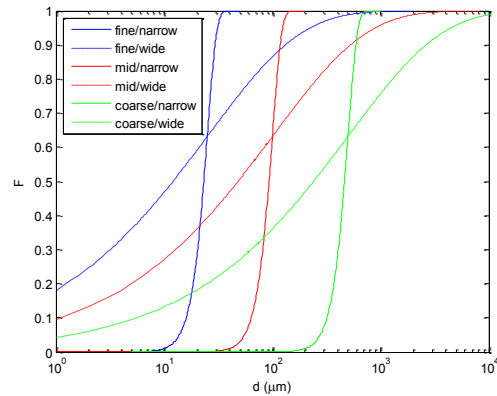


Fig. 1. Cumulative particle size distributions of the six cases considered

Several further parameters describing the flow, the transported solid, the transporting system and the cost are kept constant as follows:

- Flow and material parameters: $\mu = 1.003 \times 10^{-3}$ Pa·s ;
 $\rho_L = 1000$ kg/m³; $S = 2.82$; $\rho_s = 2820$ kg/m³; $B_0 = 0.454$;
 $B_1 = 0.0656$; $Q_s = 10$ kg/s; $W_i = 18$ kWh/ton

- Parameters of transporting system: $\gamma_p = 78480 \text{ N/m}^3$; $\varepsilon_0 = 0.05 \text{ mm}$; $C_c = 0.12$; $\eta_w = 0.6$; $ER = 0.95$; $\eta_m = 0.9$; $P = 80 \text{ kW}$
- Cost parameters: $C_{1,1} = 1.4 \text{ £/W/year}$; $C_2 = 0.1 \text{ £/N}$

Two time-dependent parameters, the pipe roughness and the levelized net annual cost of the pumping energy, are assumed constant in each 5-year interval. Values for each 5-year interval are obtained for the pipe roughness from the initial roughness height and the roughness growth rate, whereas for the levelized net annual cost of the pumping energy from $C_{1,1}$ and assuming 3% of inflation.

3.1. Life-cycle cost analysis without crushing of particles

The life-cycle cost analysis is carried out considering solid with the six weighted mean diameters given in Table 1. For each particle size, the lifetime of the pipeline is varied by 5-year increments, and the optimum diameter is determined for each lifetime so that the total cost per unit pipe length per unit volume of transported mixture throughout the assumed lifetime is minimum. The annual total cost per unit length of the pipe is also calculated for each lifetime, and the case when it is minimum provides the optimum lifetime with the corresponding optimum diameter. Finally, the pipe length is determined for this case as the ratio of the known pumping power and the calculated pumping power per unit pipe length.

The optimum lifetimes considering the different particle sizes are shown in Table 2. The optimum lifetime decreases with particle size, which is a consequence of the fact that transportation of coarser solid causes faster pipe wear.

Table 2. Optimum lifetime of pipeline for pipes transporting solids with different particle size distributions and flow parameters as obtained for a pipe with optimum diameter operating throughout optimal lifetime (d – weighted mean diameter, t_T – optimum lifetime, V – flow velocity, C_v – volume fraction of solid, C_w – mass fraction of solid, Q – total volumetric flow rate)

d (μm)	t_T (year)	V (m/s)	C_v	C_w	Q (lit/s)
23	20	1.57	0.02	0.05	185
50	10	1.53	0.05	0.13	71
92	10	1.51	0.11	0.25	33
200	5	1.65	0.11	0.26	32
460	5	2.17	0.03	0.09	110
1000	5	2.97	0.01	0.02	547

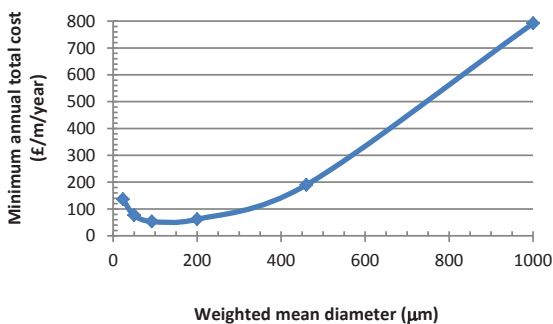


Fig. 2. Annual total cost per unit pipe length as a function of particle size of the transported solid for pipes operating throughout the optimum lifetime

The variation of minimum annual total cost per unit pipe length with particle size is provided in Fig. 2. For each particle size, the points in this curve represent the lowest annual cost that is obtained assuming the optimum lifetime given in Table 2. Among the minimum annual costs, the lowest values are obtained for mid-size solids (weighted mean diameter of 100-200 μm). The annual total cost may be as low as 50 £/m/year .

The optimum diameter is also smallest when transporting mid-size solid (see Fig. 3). In this case, the optimum diameter is about 15 cm. If the solid is very fine or very coarse (extremities of the curve in Fig. 3), then the analysis results in a so big pipe that is not practical to use. This result may be caused by the fact that when the parameter K in the Durand equation was determined, solids with particle size in the range of several 10 μm to several 100 μm were used. Thus, when the particle size approaches 10 μm or 1 mm, then the equation with the K determined is not applicable.

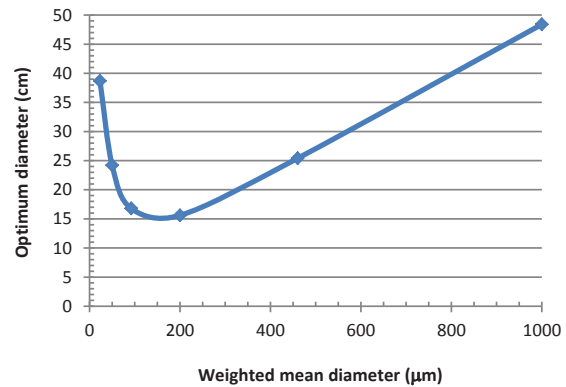


Fig. 3. Optimum diameter as a function of particle size of the transported solid for pipes operating throughout the optimum lifetime

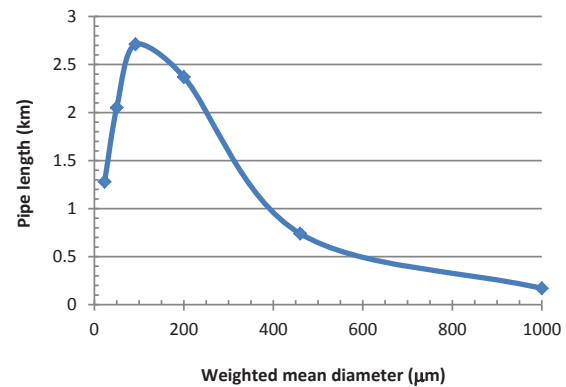


Fig. 4. Pipe length between two successive pumping units for a pumping power of 80 kW as a function of particle size of the transported solid for pipes with optimum diameter operating throughout the optimum lifetime

Fig. 4 shows the pipe lengths between two successive pumping units for a pumping power of 80 kW when solids with different particle sizes are transported. The longest pipe length (close to 3 km) is obtained for mid-size solids. In other

words, this is the case when the lowest number of pumping units must be installed along the entire pipeline.

Table 2 also shows values of flow parameters obtained for the pipeline with optimum diameter operating throughout the optimum lifetime. Similarly to optimum diameter, values of concentration and flow rate fall out of range used in practice for very fine and very coarse solids. However, apart from the extremities, all of the flow parameters fall in ranges of practical use.

3.2. Cost reduction by crushing of particles

The lowest annual total cost is obtained for the mid-size solids where particle size is spread in a narrow region (“mid-size/narrow” distribution in Table 1). Thus, it may be economical to crush the solid particles to obtain this distribution and transport these particles when the particle size in a solid is greater than that in the “mid-size/narrow” distribution. Since the representative diameter d_{80} is used in the Bond crushing law (see Section 2.4), it is presented in Table 3 together with the results obtained. The diameter d_{80} of the “mid-size/narrow” distribution is 110 μm .

Table 3. Results of life-cycle cost analysis with and without crushing (d_{80} – representative diameter, L_{total} – total length of pipeline, $t_{r,opt}$ – optimal lifetime, D_{opt} – optimal pipe diameter, C_{tot} – minimum annual total cost, “no cr.” – without crushing)

Solid / size region	d_{80} (μm)	L_{total} (km)	$t_{r,opt}$ (yr)	D_{opt} (cm)	C_{tot} (£/m/yr)
Mid-size / wide	260	300	10	18.0	63.83
Mid-size / wide	260	no cr.	5	12.3	61.72
Mid-size / wide	260	400	10	17.7	61.15
Coarse / narrow	550	20	10	28.8	225.44
Coarse / narrow	550	no cr.	5	17.7	189.67
Coarse / narrow	550	30	10	26.3	175.32
Coarse / wide	1295	4	10	46.8	908.72
Coarse / wide	1295	no cr.	5	28.7	791.97
Coarse / wide	1295	5	10	44.0	757.68

The contribution of crushing to the total cost is considerable when the total length of pipeline is short, and it becomes negligible for very long pipelines. Thus, if the distance between the site of mining process and the destination of transportation is known, the life-cycle cost analysis can determine whether crushing before transportation is economical. Table 3 shows that crushing of the “mid-size/wide” distribution used in this study is economical only if the total length of pipeline is at least 400 km. This is a consequence of the fact that the annual total cost of transportation of a solid with such particle size distribution is hardly greater than that of a solid with “mid-size/narrow” distribution. Thus, crushing becomes economical only for quite long pipelines. However, the annual total cost of transportation of coarse solids is significantly higher; therefore crushing becomes economical for much shorter pipelines (e.g. 30 km and 5 km, respectively, for solids with “coarse/narrow” and “coarse/wide” distributions used in this study).

4. Conclusions

A life-cycle cost analysis model developed in this study is applied to examine the effects of particle size distribution of the solid particles to be transported on optimal sizing and lifetime of pipelines used for transportation of solid-liquid mixtures. The optimum lifetime is obtained when the annual total cost per unit length of the pipe is minimum, whereas the optimum diameter corresponds to the least total cost per unit pipe length per unit volume of transported mixture throughout this lifetime. The pipe length between two successive pumping units may also be calculated if the pumping power is known beforehand. The effects of particle size may be summarized as follows:

- The optimum lifetime decreases when the transported solid is coarser.
- The minimum annual total cost (i.e. the annual total cost throughout the optimum lifetime) is minimum for mid-size solids (weighted mean diameter of 100-200 μm). This cost can be as low as 50 £/m/yr , and it may increase an order of magnitude when carrying coarse solid particles.
- The optimum diameter is about 15 cm for mid-size solids, and increases for finer and coarser solids.
- The pipe length between two successive pumping units for a pumping power of 80 kW approaches 3 km for mid-size solids, and decreases for finer and coarser solids.
- The method is applicable to determine whether crushing before transportation is economical. When the solid is coarser, crushing becomes economical for a shorter distance between the site of mining process and destination of transportation.

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