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Accuracy/Speed Analysis of Pipe Friction Factor Correlations*

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Abstract. The Colebrook [1] equation is considered the standard for the calculation of friction factor for turbulent flow in commercial pipes, but it is implicit, and therefore it must be computed by iterative methods. Although such iterative computation quickly converges, the computational time in large pipe system simulations can be reduced using an accurate explicit correlation. A review of the up to date literature identified 30 different explicit correlations. In order to determine which correlation is the best alternative to Colebrook's, both accuracy and computational burden were compared. The accuracy of each explicit correlation was compared against Colebrook's correlation using the mean and maximum relative errors and the coefficient of determination. Also, the computational time of each equation was measured using the tic and toc functions in GNU Octave software. It was found that the iterative computation of the Colebrook equation demands about 2.6 times the computational time of the slowest explicit correlation. The correlations with the best balance between accuracy and computational burden are, in decreasing order of accuracy and increasing order of speed, correlations by Serghides [13] (Eqs. (17), (18), (19), and (20)), by Shacham [8] (Eqs. (10) and (11)), by Brkić and Praks [33] (Eqs. (53), (54), (55), and (56)), and by Fang et al. [19] (Eq. (28)).

Keywords: pipe flow, friction factor, Colebrook, correlations

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1 Introduction

Fluid flow in pipes and ducts is widely used in applications such as the transportation of oil and gas, irrigation, water distribution, air conditioning, and power plants. The friction factor is used in the computation of head loss or pressure drop in pipes, which is the loss of the flow's mechanical energy due to viscous friction. Major or distributed losses are caused by viscous friction caused by the pipe's wall while minor or local losses are caused by the recirculating and turbulent mixture of fluid caused by inlets, outlets, contractions, expansions, curves, valves, junctions, among others structural features of the pipe through the flow's path. These losses h_L are defined as:

$$h_L = f \frac{L}{D} \frac{\overline{V}^2}{2g} \tag{1}$$

where f is the friction factor, L is the pipe's length, D is the pipe's internal diameter, $\overline{V}$ is the average fluid velocity, and g is the gravitational acceleration. For minor losses, an equivalent pipe length can be used.

For laminar flow, the friction factor is computed analytically as a function of the Reynolds number, Re, and for pipes of circular section it is defined as:

$$f = \frac{64}{\mathrm{Re}} \tag{2}$$

where Re is defined as:

$$\mathrm{Re} = \frac{\rho \overline{V} D}{\mu} \tag{3}$$

where ρ is the fluid's specific mass and μ is the fluid's dynamic viscosity. For transition and turbulent flow in commercial pipes, Colebrook [1] proposed, in 1939, Equation (4) based on experimental data.

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}} \right) \tag{4}$$

where ϵ is the absolute internal roughness of the pipe and ϵ/D is its relative roughness. Eq. (4) is considered the standard in friction factor computation, but it is implicit, and therefore it must be computed by some iterative method. In 1944, Moody [3] plotted this equation in a diagram to simplify its usage in engineering. Also, many explicit correlations were proposed since then, mostly as approximations of Colebrook's implicit correlation. Although the iterative computation converges quickly, computational time in large pipe system simulations can be reduced using a sufficiently accurate explicit correlation.

In this context, the objective of this work is to make a comparative analysis between the Colebrook equation and explicit correlations available in the literature. The accuracy, against the Colebrook equation, and the computational time of the correlations are compared. To check the accuracy of Colebrook's correlation relative to experimental data in real pipes is beyond the objective of this work. It should be mentioned that the application of any correlation in real pipe systems depends on measurements of relative roughness and Reynolds number, and therefore they propagate intrinsic uncertainties to the friction factor [2].

2 Explicit Correlations

Since Colebrook put forward his equation, many explicit correlations for turbulent friction factor were proposed. Here, the chronological appearance of 30 different explicit equations is depicted from Eq. (5), which was proposed by Moody [4] himself in 1947, to Brkić and Praks' [33] in 2019.

$$f = 0.0055\left[1 + \left(2\times 10^4\frac{\epsilon}{D} + \frac{10^6}{\mathrm{Re}}\right)^{1/3}\right] \tag{5}$$

In 1966, Wood [5] suggested Eq. (6). It should be noted that this equation is not valid to relative roughness equal to zero.

$$f = 0.53\frac{\epsilon}{D} + 0.094\left(\frac{\epsilon}{D}\right)^{0.225} + 88\left(\frac{\epsilon}{D}\right)^{0.44}\mathrm{Re}^{A_1} \tag{6}$$

where:

$$A_1 = -1.62\left(\frac{\epsilon}{D}\right)^{0.134} \tag{7}$$

Eq. (8) was proposed by Churchill [6] in 1973.

$$f = \left\{-2\log_{10}\left[\frac{\epsilon/D}{3.7} + \left(\frac{7}{\mathrm{Re}}\right)^{0.9}\right]\right\}^{-2} \tag{8}$$

Swamee and Jain [7] introduced Eq. (9) in 1976.

$$f = \frac{0.25}{\left\{\log_{10}\left[(\epsilon/D)/3.7 + 5.74/\mathrm{Re}^{0.9}\right]\right\}^2} \tag{9}$$

Eq. (10) was recommended by Shacham [8] in 1980.

$$f = \left\{\left[A_2(1-\ln A_2) - \frac{\epsilon/D}{3.7}\right]\Big/\left(1.15129A_2 + \frac{2.51}{\mathrm{Re}}\right)\right\}^{-2} \tag{10}$$

where:

$$A_2 = \frac{\epsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}}\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{14.5}{\mathrm{Re}}\right) \tag{11}$$

In 1981, Eq. (12) was suggested by Barr [9].

$$f = \left(-2\log_{10}\left\{\frac{\epsilon/D}{3.7} + \frac{4.518\log_{10}(\mathrm{Re}/7)}{\mathrm{Re}\left[1 + \left(\mathrm{Re}^{0.52}/29\right)(\epsilon/D)^{0.7}\right]}\right\}\right)^{-2} \tag{12}$$

Zigrang and Sylvester [10] proposed, in 1982, Eq. (13).

$$f = \left\{-2\log_{10}\left[\frac{\epsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}}\log_{10}\left(\frac{\epsilon/D}{3.7} - \frac{5.02}{\mathrm{Re}}\log_{10} A_3\right)\right]\right\}^{-2} \tag{13}$$

where:

$$A_3 = \frac{\epsilon/D}{3.7} + \frac{13}{\mathrm{Re}} \tag{14}$$

In 1983, Haaland [11] submitted Eq. (15).

$$f = \left\{-1.8\log_{10}\left[\frac{6,9}{\mathrm{Re}} + \left(\frac{\epsilon/D}{3,7}\right)^{1.11}\right]\right\}^{-2} \tag{15}$$

In 1984, Chen [12] recommended Eq. (16).

$$f = 0.3164\left(\frac{1}{\mathrm{Re}^{0.83}} + 0.11\frac{\epsilon}{D}\right)^{0.3} \tag{16}$$

Eq. (17) was proposed by Serghides [13] in 1984.

$$f = \left[A_4 - \frac{(B_4 - A_4)^2}{A_4 - 2B_4 + C_4}\right]^{-2} \tag{17}$$

where:

$$A_4 = -2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{12}{\mathrm{Re}}\right) \tag{18}$$

$$B_4 = -2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51A_4}{\mathrm{Re}}\right) \tag{19}$$

$$C_4 = -2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51A_4}{\mathrm{Re}}\right) \tag{20}$$

In 1997, Manadilli [14] presented Eq. (21).

$$f = \left[-2\log_{10}\left(\frac{\epsilon/D}{3.70} + \frac{95}{\mathrm{Re}^{0.983}} - \frac{96.82}{\mathrm{Re}}\right)\right]^{-2} \tag{21}$$

Sonnad and Goudar [15], in 2006, proposed Eq. (22).

$$f = \left\{0.8686\ln\left[\frac{0.4587\mathrm{Re}}{A_5^{A_5/(A_5+1)}}\right]\right\}^{-2} \tag{22}$$

where:

$$A_5 = 0.124\frac{\epsilon}{D}\mathrm{Re} + \ln(0.4587\mathrm{Re}) \tag{23}$$

Vatankhah [16], in 2014, optimized Eq. (22). Equation (24) is the optimized form.

$$f = \left\{0.8686 \ln \left[\frac{0.4599\text{Re}}{(A_6 - 0.2753)^{A_6/(A_6+0.9741)}}\right]\right\} \tag{24}$$

where:

$$A_6 = 0.124\frac{\epsilon}{D}\text{Re} + \ln(0.4599\text{Re}) \tag{25}$$

Avci and Karagoz [17] suggested, in 2009, Eq. (26).

$$f = \frac{6.4}{\left\{\ln \text{Re} - \ln\left[1 + 0.01\text{Re}(\epsilon/D)\left(1 + 10\sqrt{\epsilon/D}\right)\right]\right\}^{2.4}} \tag{26}$$

Eq. (27) was introduced by Papaevangelou et al. [18] in 2010.

$$f = \frac{0.2479 - 0.0000947(7 - \log_{10}\text{Re})^4}{\left\{\log_{10}\left[(\epsilon/D)/3.615 + 7.366/\text{Re}^{0.9142}\right]\right\}^2} \tag{27}$$

In 2011, Fang et al. [19] recommended Eq. (28).

$$f = 1.613\left\{\ln\left[0.234\left(\frac{\epsilon}{D}\right)^{1.1007} - \frac{60.525}{\text{Re}^{1.1105}} + \frac{56.291}{\text{Re}^{1.0712}}\right]\right\}^{-2} \tag{28}$$

Ghanbari et al. [20] proposed, in 2011, Eq. (29).

$$f = \left\{-1.52\log_{10}\left[\left(\frac{\epsilon/D}{7.21}\right)^{1.042} + \left(\frac{2.731}{\text{Re}}\right)^{0.9152}\right]\right\}^{-2.169} \tag{29}$$

Eq. (30) was submitted by Samadianfard [21] in 2012.

$$f = \left[\frac{\text{Re}^{\epsilon/D} - 0.6315093}{\text{Re}^{1/3} + \text{Re}(\epsilon/D)}\right] + 0.0275308\left(\frac{6.929841}{\text{Re}} + \epsilon/D\right)^{1/9} + A_7 \tag{30}$$

where:

$$A_7 = \left[\frac{10^{\epsilon/D}}{(\epsilon/D) + 4.781616}\right]\left(\sqrt{\epsilon/D} + \frac{9.99701}{\text{Re}}\right) \tag{31}$$

In 2014, Vatankhah [16] also proposed Eq. (32).

$$f = \left[\frac{2.51/\text{Re} + 1.1513A_8}{A_8 - (\epsilon/D)/3.71 - 2.3026A_8\log_{10}(A_8)}\right]^2 \tag{32}$$

where:

$$A_8 = \frac{6.0173}{\mathrm{Re}\big[0.07(\epsilon/D) + \mathrm{Re}^{-0.885}\big]^{0.109}} + \frac{\epsilon/D}{3.71} \tag{33}$$

In 2015, Heydari et al. [22] proposed Eq. (34).

$$f = \left[0.42 + 16.27\frac{\epsilon}{D} - 1.81\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right) + A_9 + B_9\right]^{-2} \tag{34}$$

where:

$$A_9 = -54.81(\epsilon/D)^2 + 0.02\left[\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2 \tag{35}$$

$$B_9 = 8.74(\epsilon/D)\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{5.74}{\mathrm{Re}^{0.9}}\right) \tag{36}$$

Eq. (37) was suggested by Offor and Alabi [23] in 2016.

$$f = \left(-2\log_{10}\left\{\frac{\epsilon/D}{3.71} - \frac{1.975}{\mathrm{Re}}\ln\left[\left(\frac{\epsilon/D}{3.93}\right)^{1.092} + \frac{7.627}{\mathrm{Re} + 395.9}\right]\right\}\right)^{-2} \tag{37}$$

In 2016, Beluco and Schettini [24] presented Eq. (38).

$$f = 0.3009\left\{\log_{10}\left[\left(\frac{\epsilon/D}{3.7315}\right)^{1.0954} + \left(\frac{5.9802}{\mathrm{Re}}\right)^{0.9695}\right]\right\}^{-2} \tag{38}$$

In 2017, Biberg [25] recommended Eq. (39).

$$f = \left\{\frac{2}{\ln 10}\left[\ln\left(\frac{\ln 10\ \mathrm{Re}}{5.02}\right) + \left(\frac{1}{A_{10}} - 1\right)\ln A_{10}\right]\right\}^{-2} \tag{39}$$

where:

$$A_{10} = \ln\left(\frac{\ln 10\ \mathrm{Re}}{5.02}\right) + \frac{\ln 10\ \mathrm{Re}}{18.574}\frac{\epsilon}{D} \tag{40}$$

In 2017, Brkić and Ćojbašić [26] suggested optimized forms of various explicit correlations. Their goal was to minimize the maximum relative error of these equations. Eq. (41) is the optimized form of the correlation proposed by Eck [27] in 1966 and Eq. (42) is the optimized form of the correlation introduced by Chen [28] in 1979.

$$f = \left[-1.963\log_{10}\left(\frac{14.064}{\mathrm{Re}} + \frac{\epsilon/D}{4.034}\right)\right]^{-2} \tag{41}$$

$$f = \left(-2.003\log_{10}\left\{\frac{\epsilon/D}{3.689} - \frac{4.933}{\mathrm{Re}}\log_{10}\left[\frac{1}{2.762}\left(\frac{\epsilon}{D}\right)^{1.109} + A_{11}\right]\right\}\right)^{-2} \tag{42}$$

where:

$$A_{11} = \frac{5.89}{\mathrm{Re}^{0.923}} \tag{43}$$

Eq. (44) is the optimized form of Round's [29] equation from 1980.

$$f = \left\{1.898\log_{10}\left[\frac{\mathrm{Re}}{0.202\mathrm{Re}(\epsilon/D) + 9.779}\right]\right\}^{-2} \tag{44}$$

Eq. (45) is the optimized form of correlation by Romeo et al. [30] from 2002.

$$f = \left\{-2\log_{10}\left[\frac{\epsilon/D}{3.7106} - \frac{5}{\mathrm{Re}}\log_{10}\left(\frac{\epsilon/D}{3.8597} - \frac{4.795}{\mathrm{Re}}\log_{10} A_{12}\right)\right]\right\}^{-2} \tag{45}$$

where:

$$A_{12} = \left(\frac{\epsilon/D}{7.646}\right)^{0.9685} + \left(\frac{4.9755}{206.2795 + \mathrm{Re}}\right)^{0.0,8759} \tag{46}$$

Eq. (47) is the optimized form of the equation proposed by Buzzelli [31] in 2008.

$$f = \left\{A_{13} - \left[\frac{A_{13} + 1.9999\log_{10}(B_{13}/\mathrm{Re})}{0.9996 + 2.1018/B_{13}}\right]\right\}^{-2} \tag{47}$$

where:

$$A_{13} = \frac{0.7314\ln\mathrm{Re} - 1.3163}{1.0025 + 1.2435\sqrt{\epsilon/D}} \tag{48}$$

$$B_{13} = \frac{\epsilon/D}{3.7165}\mathrm{Re} + 2.5137A_{13} \tag{49}$$

Eq. (50) was introduced by Gregory and McEnery [32] in 2017.

$$f = \left[-2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{1.64A_{14}}{\mathrm{Re}^{B_{14}}}\right)\right]^{-2} \tag{50}$$

where:

$$A_{14} = 0.9 - 4.9\frac{\epsilon}{D} + 0.1e^{-400(\epsilon/D)} \tag{51}$$

$$B_{14} = 1 - \frac{1}{4 + 0.208\ln(\mathrm{Re}/3000)} \tag{52}$$

In 2019, Brkić and Praks [33] submitted Eq. (53).

$$f = \left\{0.8686 \left[A_{15} - C_{15} + \frac{C_{15}}{A_{15} + B_{15}}\right]\right\}^{-2} \tag{53}$$

where:

$$A_{15} = \ln \mathrm{Re} - 0.779397488 \tag{54}$$

$$B_{15} = \frac{\mathrm{Re}(\epsilon/D)}{8.0878} \tag{55}$$

$$C_{15} = \ln\left(A_{15} + B_{15}\right) \tag{56}$$

This list is not exhaustive, but it contains the explicit correlations that are frequently referenced in similar accuracy and complexity comparison works [34–38]. Some correlations were excluded because the required reference was not available, due to minor changes from the presented ones, or due to the excessive complexity of their equations.

3 Methodology

The evaluation of accuracy, against the Colebrook equation, and computational burden of the explicit correlations presented in Section 2 were performed in GNU Octave software, version 4.2.1, running in a computer with an Intel Core i5−6600 3.90 GHz processor and 8 GB of RAM.

3.1 Accuracy Evaluation

The accuracy of the 30 explicit correlations from Eq. (5) to (56) was evaluated by their range of error (minimum and maximum errors), average error and coefficient of determination or R^2, all against the Colebrook equation.

A domain of 402 values of relative roughness in $10^{-6} \leq \epsilon/D \leq 0.05$ plus $\epsilon/D = 0$ (0, 1.0×10^{-6}, 1.1×10^{-6}, ..., 9.9×10^{-6}, 1.0×10^{-5}, 1.1×10^{-5}, ..., 5.0×10^{-2}) by 4201 values of Reynolds number in $4000 \leq \mathrm{Re} \leq 10^8$ (4.00×10^3, 4.01×10^3, ..., 9.99×10^3, 1.00×10^4, 1.01×10^4, ..., 1.00×10^8) was created yielding a total of 1688802 points, corresponding to the ranges of values used by Moody [3] in his diagram. The friction factor was calculated for all points by the Colebrook equation and by each explicit correlation. Smooth pipe, i.e. $\epsilon/D = 0$, was not evaluated in Wood's [5] correlation (Eqs. (6) and (7)) because it does not accept this value. The Colebrook equation was calculated by the fixed point iterative method defined as:

$$\frac{1}{\sqrt{f_{i+1}}} = -2\log_{10}\left(\frac{\epsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f_i}}\right), i = 1, 2, \ldots, N \tag{57}$$

where f_i is the friction factor in the iteration i, f_{i+1} is the friction factor to be used in the next iteration, and N is the number of iterations. An initial guess is used for f_i in the first iteration, and it is followed by the iterations until the criterion of convergence is achieved. The initial value used was 0.0425, which is the average of the minimum and maximum friction factor values in the Moody diagram, i.e. 0.005 and 0.08. The criterion of convergence was an absolute error between two consecutive iterations of less than 10^{-10}. Using this iterative method, Colebrook's correlation converges after an average of 8 iterations. Also, the initial value does not affect too much the convergence as long as it is larger than zero.

The relative dimensionless error ε of each point is defined as:

$$\varepsilon = \frac{f_C - f_{CW}}{f_{CW}} \tag{58}$$

where f_C is the friction factor calculated by the correspondent explicit correlation and f_{CW} is the friction factor calculated by the Colebrook equation. The average of the relative error $\overline{\varepsilon}$ is defined as:

$$\overline{\varepsilon} = \frac{1}{P}\sum_{i=1}^{P} \frac{|f_{C_i} - f_{CW_i}|}{f_{CW_i}} \tag{59}$$

where P is the number of points used in the accuracy evaluation, i.e. 1688802, f_{C_i} is the friction factor calculated by the correspondent explicit correlation for the point, and f_{CW_i} is the friction factor calculated by Colebrook's correlation for the point. Also, the ratio of maximum relative error was calculated as the correlation's maximum relative error divided by the smallest maximum relative error among all correlations.

The coefficient determination R^2 is defined as:

$$R^2 = 1 - \sum_{i=1}^{P} \left(f_{CW_i} - f_{C_i}\right)^2 / \sum_{i=1}^{P} \left(f_{CW_i} - \overline{f}_{CW}\right)^2 \tag{60}$$

where $\overline{f}_{CW}$ is the average of all values of friction factor calculated by the Colebrook equation, which is approximately 0.025, defined as:

$$\overline{f}_{CW} = \frac{1}{P}\sum_{i=1}^{P} f_{CW_i} \tag{61}$$

3.2 Computational Burden Evaluation

The computational times of the 30 explicit correlations from Eq. (5) to (56) and of the Colebrook equation were measured using the tic and toc functions in GNU Octave software release 4.2.1.

It was used a domain of 42 values of relative roughness in $10^{-6} \leq \epsilon/D \leq 0.05$ plus $\epsilon/D = 0$ by 421 values o Reynolds number in $4000 \leq \mathrm{Re} \leq 10^8$ with a total of 17682 points. The friction factor was calculated by the Colebrook equation and by each explicit correlation for all points, and the time taken was measured.

As in the accuracy evaluation, smooth pipe, i.e. $\epsilon/D = 0$, was not evaluated in Eqs. (6) and (7), which correspond to the correlation proposed by Wood [5]. Also, the Colebrook equation was calculated by the fixed point iterative method with 0.0425 as an initial value and an absolute error between two consecutive iterations of less than 10^{-10} was used as the criterion of convergence. This computational time measurement was repeated nine times and were calculate their average and standard deviation σ:

$$\sigma = \sqrt{\frac{1}{M-1}\sum_{i=1}^{M}\left(t_i - \bar{t}\right)^2} \tag{62}$$

where M is the number of measurements, i.e. 9, t_i is the computational time measured and $\bar{t}$ is the average of the measured computational times. Also, in order to make this computational time dimensionless and independent of the computational power of the computer, the ratio of computational time was calculated as the correlation's computational time divided by the smallest computational time among all correlations.

4 Results and Discussion

The results from both accuracy and computational time evaluations are summarized and discussed in this section. The computational codes developed in this work are published in the Code Ocean platform and available in: https://codeocean.com/capsule/7657118/.

4.1 Accuracy of the Correlations

Table 1 shows the accuracy of the explicit correlations. Their ratio of maximum error, range of error, average error and R^2 are shown. All 30 explicit correlations are ranked from the smallest to the largest maximum relative error, in absolute values. The ranking does not change significantly if the average error is used as the main criteria of comparison.

Correlation by Serghides [13] (Eqs. (17) to (20)) is the most accurate one, with maximum relative error of 0.00314%. The next is correlation by Shacham [8] (Eq. (10) and (11)), with maximum relative error of 0.01740%. With maximum relative errors between 0.10407% and 0.15285% are correlations by Buzzelli [26] (Eqs. (47) to (49)), by Zigrang and Sylvester [10] (Eqs. (13) and (14)), by Brkić and Praks [33] (Eqs. (53) to (56)), by Offor and Alabi [23] (Eq. (37)), by Vatankhah [16] (Eqs. (32) and (33)), by Romeo et al. [26] (Eqs. (45) and (46)), by Sonnad and Goudar [16] (Eqs. (24) and (25)) and by Biberg [25] (Eqs. (39) and (40)). All these correlations have average error equal or less than 0.05500% and R^2 equal to 1.00000.

Correlations by Chen [26] (Eq. (43)), by Fang et al. [19] (Eq. (28)), by Barr [9] (Eq. (12)), by Papaevangelou et al. [18] (Eq. (27)) and by Sonnad and Goudar [15] (Eqs. (22) and (23)) have maximum relative errors between

Table 1. Ranking of the most to the less accurate explicit correlations. Error ratio is the maximum error of the correlation divided by the smallest maximum error among all correlations, ε is the range of errors, $\overline{\varepsilon}$ is the average error in absolute value and R^2 is the coefficient of determination.

Equation(s)	Error ratio	ε [%]	$\overline{\varepsilon}$ [%]	R^2
(17), (18), (19), and (20)	1.00000	[-0.00314, 0.00000]	0.00055	1.00000
(10) and (11)	5.54637	[-0.00044, 0.01740]	0.00206	1.00000
(47), (48), and (49)	33.16356	[-0.10407, 0.07123]	0.03280	1.00000
(13) and (14)	36.20211	[-0.11360, 0.04060]	0.02447	1.00000
(53), (54), (55), and (56)	39.66605	[-0.12447, 0.08703]	0.05500	1.00000
(37)	39.93023	[-0.12530, 0.04924]	0.05199	1.00000
(32) and (33)	39.93048	[-0.12530, 0.05958]	0.04297	1.00000
(45) and (46)	42.31998	[-0.13280, 0.00137]	0.04521	1.00000
(24) and (25)	44.33863	[-0.13913, -0.00509]	0.04811	1.00000
(39) and (40)	48.71029	[-0.10478, 0.15285]	0.03583	1.00000
(42) and (43)	96.82851	[-0.30384, 0.10366]	0.12043	0.99999
(28)	156.62707	[-0.44092, 0.49149]	0.16294	0.99999
(12)	170.10547	[-0.53378, 0.32094]	0.06569	0.99999
(27)	257.77584	[-0.80889, 0.57827]	0.17521	0.99998
(22) and (23)	316.34152	[0.00187, 0.99267]	0.25125	0.99993
(34), (35), and (36)	417.81326	[-1.25486, 1.31108]	0.67725	0.99984
(50), (51), and (52)	431.85845	[-1.35515, 0.72541]	0.22497	0.99997
(15)	453.68782	[-1.42365, 1.31384]	0.44954	0.99991
(21)	869.52101	[-0.00407, 2.72852]	0.38169	0.99972
(29)	922.96631	[-2.89623, 2.15521]	0.72309	0.99937
(26)	954.27655	[-2.99448, 2.90109]	1.03591	0.99901
(38)	1046.80961	[-0.93384, 3.28485]	0.28874	0.99997
(9)	1070.18734	[-0.70862, 3.35820]	0.51644	0.99959
(8)	1089.57825	[-0.62086, 3.41905]	0.52977	0.99957
(44)	1742.94614	[-5.15812, 5.46929]	2.31334	0.99863
(41)	2545.82949	[-6.94966, 7.98871]	2.11387	0.99685
(30) and (31)	3956.45491	[-12.41519, 7.42849]	1.75428	0.99889
(5)	5066.56332	[-15.89867, 12.53223]	3.09869	0.97956
(6) and (7)	8997.41867	[-28.23353, 6.24061]	3.30019	0.99387
(16)	14578.89625	[-45.74798, 10.37852]	8.63559	0.98286

0.30384% and 0.99267%. The correlations by Heydari et al. [22] (Eqs. (34) and (35)), by Gregory and McEnery [32] (Eqs. (50) to (52)), and by Haaland [11] (Eq. (15)) have, respectively, 1.31108%, 1.35515%, and 1.42365% maximum relative errors.

Correlations by Manadilli [14] (Eq. (21)), by Ghanbari et al. [20] (Eq. (29)), by Avci and Karagoz [17] (Eq. (26)), by Beluco and Schettini [24] (Eq. (38)), by Swamee and Jain [7] (Eq. (9)), by Churchill [6] (Eq. (8)), and by Round [26] (Eq. (44)) have maximum relative errors between 2.72852% and 5.46929%.

With maximum relative error equal or greater than 7.98871% are correlations by Eck [26] (Eq. (41)), by Samadianfard [21] (Eqs. (30) and (31)), by Moody [4] (Eq. (5)), by Wood [5] (Eqs. (6) and (7)) and by Chen [12] (Eq. (16)). Among the explicit correlations analysed, equation by Chen has the greatest maximum relative error of 45.74798%, the greatest average relative error of 8.63559% and the least R^2 of 0.98286.

4.2 Computational Burden of the Correlations

Table 2 shows the computational time of the explicit and Colebrook's correlations. Their time ratio, average and standard deviation of computational time are presented. All correlations are ranked from the smallest to the largest average computational time.

As expected, in general, the fastest correlations are also the least accurate ones. The most accurate explicit correlation (Eqs. (17) to (18)) has computational time 4.5 times greater than the fastest (Eq. (5)). One important result is that the fixed point iterative method of calculation of the Colebrook equation, which is Eq. (57), is the most time consuming one. Its computational time is 2.6 times longer than the slowest explicit correlation (Eqs. (34) to (36)).

4.3 Speed/Accuracy Analysis of the Correlations

The computational time of Colebrook's correlation is larger than those of the explicit correlations. Therefore, depending on the accuracy need, any explicit correlation can be used to reduce the computational time in simulations. Figure 1 shows the plot of the explicit correlations' ratio of maximum relative error (in the abscissa axis) by their ratio of computational time (in the ordinate axis) and allows to determine the explicit correlations with the best balance between accuracy and computational burden.

The points marked with squares are the correlations with the best balance between accuracy and computational time. The shorter computational time and lower maximum relative error, the better. Points closer to the origin (0,0) or to either axis mean better explicit correlations. The first one, in the top left, is Serghides' [13] correlation (Eqs. (17) to (20)), with maximum error of 0.00314% and ratio of time of 4.47833. Its more accurate and less time consuming than the correlation by Heidary et al. [22] (Eqs. (34) and (35)), which is the point in the top of Figure 1.

Table 2. Ranking of the fastest to the slowest correlations. Time ratio is the average computational time of the correlation divided by the smallest average computational time among all correlations, $\bar{t}$ is the average computational time and σ is its standard deviation.

Equation(s)	Time ratio	$\bar{t}$ [s]	σ [s]
(5)	1.00000	0.23821	0.00438
(16)	1.02485	0.24413	0.01804
(28)	1.62218	0.38641	0.00509
(8)	1.62941	0.38814	0.00686
(41)	1.65171	0.39345	0.02216
(9)	1.66194	0.39589	0.01599
(38)	1.66650	0.39697	0.01020
(15)	1.70599	0.40638	0.02625
(29)	1.80823	0.43074	0.04267
(44)	2.01129	0.47910	0.05840
(21)	2.07453	0.49417	0.05694
(53), (54), (55), and (56)	2.19184	0.52211	0.00555
(6) and (7)	2.19406	0.52264	0.00486
(22) and (23)	2.31085	0.55046	0.00441
(24) and (25)	2.33074	0.55520	0.00871
(37)	2.56558	0.61114	0.01196
(26)	2.64305	0.62959	0.08425
(27)	2.87608	0.68510	0.07418
(32) and (33)	3.01924	0.71921	0.00634
(42) and (43)	3.05794	0.72842	0.03721
(50), (51), and (52)	3.20991	0.76462	0.00825
(12)	3.47848	0.82869	0.06868
(47), (48), and (49)	3.62546	0.86361	0.08607
(10) and (11)	3.62941	0.86455	0.07591
(39) and (40)	3.85815	0.91904	0.03053
(30) and (31)	3.99840	0.95245	0.00846
(13) and (14)	4.00288	0.95352	0.01309
(45) and (46)	4.09536	0.97555	0.00474
(17), (18), (19), and (20)	4.47833	1.06677	0.02110
(34), (35), and (36)	5.16117	1.22943	0.01771
(57)	13.33385	3,17622	0.02452

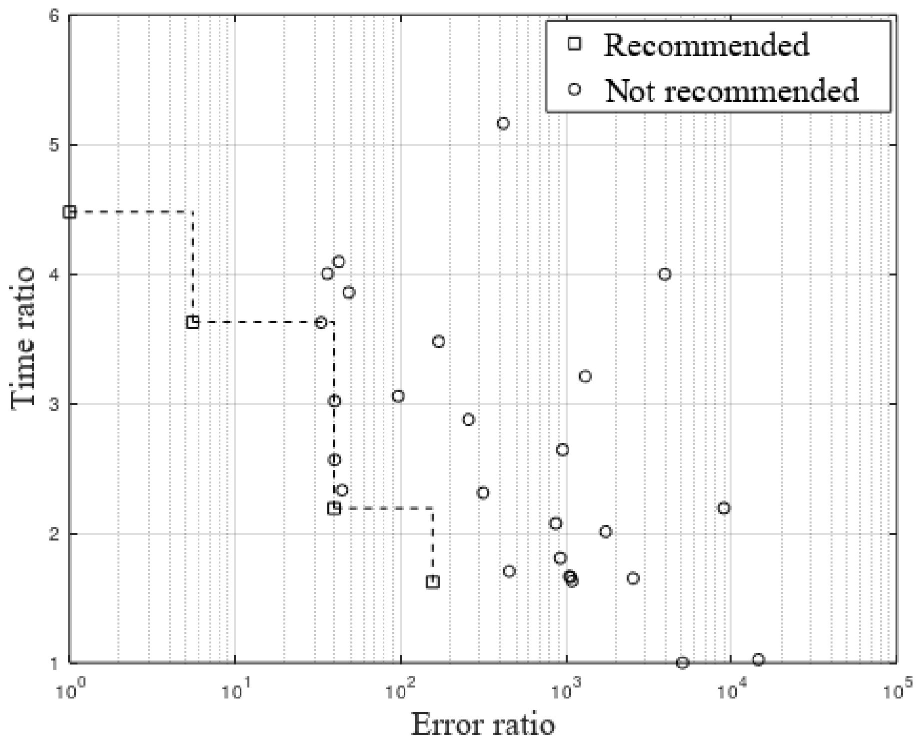

Fig. 1. Ratio of average computational time plotted against ratio of maximum relative error of the explicit correlations. The recommended explicit correlations are those with the best balance between accuracy and computational burden.

The second recommendation is correlation by Shacham [8] (Eqs. (10) and (11)), with maximum relative error of 0.01740% and ratio of time of 3.62941. Buzzelli's [26] correlation (Eqs. (47) to (49)) have a very similar ratio of time of 3.62546, but its maximum relative error is 0.10407%, which is six times greater than Shacham's.

The correlation proposed by Brkić and Praks [33] (Eqs. (53) to (56)) is the third recommendation, with maximum relative error of 0.12447% and ratio of time of 2.19406. With similar maximum error, but with slightly larger computational time, are correlations by Sonnad and Goudar [16] (Eqs. (24) and (25)), by Offor and Alabi [23] (Eq. (37)), and by Vatankhah [16] (Eqs. (32) and (33)) (on the dotted line).

The fourth and last recommendation is the correlation developed by Fang et al. [19] (Eq. (28)), with a maximum relative error of 0.49149% and ratio of time of 1.62218. Their correlation consumes about the same computational time as correlation by Haaland [11] (Eq. (15)), which has the ratio of time of 1.70599, but the relative error of the latter is up to 1.42365%. Haaland's equation is the most accurate correlation with only one input of both relative roughness and Reynolds number, and therefore it is the best suited for calculations by hand. The fastest correlation, which is the circle in the bottom of Figure 1, has error up to 15.89867%, relative to Colebrook's correlation.

5 Conclusions

The use of explicit correlations to calculated the friction factor in pipes reduces the computational time in simulations, compared to the implicit correlation of Colebrook. The slowest correlation is 2.6 times faster than Colebrook's iterative calculation for accuracy of ten decimal places. In decreasing order of accuracy and increasing order of speed, the recommend explicit correlations are: Serghides' [13] (Eqs. (17), (18), (19), and (20)) for maximum error of 0.00314% and average error of 0.00055%, Shacham's [8] (Eqs. (10) and (11)) for maximum of 0.01740% and average of 0.00206%, Brkić and Praks' [33] (Eqs. (53), (54), (55), and (56)) for maximum of 0.12477% and average 0.05500%, and Fang et al.'s [19] (Eq. (28)) for maximum error of 0.49149% and average error of 0.16294%.

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